Robust Orthogonal Fitting of Plane

Juraj GAŠINEC [1], Silvia GAŠINCOVÁ [2], Eva TREMBECZKÁ [3]

[1] Doc. Ing., Ph.D.; Institute of Geodesy, Cartography and Geographic Information Systems, FBERG, Faculty of Mining, Ecology, Process Control and Geotechnology, TUKE – Technical University of Košice, Park Komenského 19, 040 01 Košice, Slovak Republic; e-mail: juraj.gasinec@tuke.sk, tel.: +421 55 602 2846

[2] Doc. Ing., Ph.D.; Institute of Geodesy, Cartography and Geographic Information Systems, FBERG, Faculty of Mining, Ecology, Process Control and Geotechnology, TUKE – Technical University of Košice, Park Komenského 19, 040 01 Košice, Slovak Republic; e-mail: silva.gasincova@tuke.sk, tel.: +421 55 602 2846

[3] Ing. Eva Trembeczká; District Office Kosice-Cadastral Department, Južná trieda 82, 040 17 Košice, Slovak Republic; e:mail: eva.trembeczka@skgeodesy.sk, tel: (+421) 552818001

Summary

Least Squares orthogonal distance fitting of plane surface onto 3D points is the best option in the event that gross errors nor systematic errors affect the observations. However, such situations often occur in the processing of the experimental data and robust methods are a good alternative in such cases. This issue is illustrated by the example of orthogonal fitting a plane on a set of 3D points using a robust M-estimate by Huber.

Keywords: 3D scanning data, orthogonal distance fiting of plane, robust, M-estimation, outlier

Introduction

Creation of detailed spatial models of machines, their detailed parts, architectural historical monuments and artefacts, works of art, natural geological formations etc., is currently experiencing unprecedented growth due to the development of laser and optical scanning and computer technology. The results of laser and optical scanning of object are data files of 3D coordinates (Sokol, Bajtala, 2008; Franek et al. 2012), which are the basis for subsequent geometrical modelling of surfaces and solids. For many analysis, it is well-known that the surface nature of scanned object, its colour, texture, sharp edges, as well as the geometrical configuration of the scanned object and surveying equipment or other real factors may cause some measured points do not represent the scanned object. Due to the Least Squares orthogonal distance fitting of plane surface onto 3D points belong to relatively common occurring tasks, there are shown positive features of robust M–estimations on the reduction of adverse impact of 3D coordinates blunders in this paper.

General form of plane equation

As is known, any plane expressed in general form by the equation (Bartsch, 2000; Fienen, 2005):

$$ax + by + cz + d = 0\,, \tag{1}$$

where:

a, b, c –are the Cartesian components of the unit normal vector $\boldsymbol{n} = (a, b, c)^T$ (oriented perpendicular to the plane launched from the beginning of the coordinate system), complying with condition:

$$\sqrt{a^2 + b^2 + c^2} = 1 \tag{2}$$

and x, y, z are the Cartesian coordinates of a point lying on the plane (1). Estimate the unknown coefficients a, b, c is determined (Strang, 2006; Fienen, 2005) by minimizing the orthogonal distances δ_i of points $\boldsymbol{P}_i = (x_i, y_i, z_i)^T$; $i = 1, \cdots n$

$$\delta_i = \frac{ax_i + by_i + cz_i + d}{\sqrt{a^2 + b^2 + c^2}}\,. \tag{3}$$

Whereas plane (1) passes through the centroid $\boldsymbol{C} = (x_C, y_C, z_C)^T$ of all its points

$$\begin{pmatrix} x_C \\ y_C \\ z_C \end{pmatrix} = \begin{pmatrix} \sum_{i-1}^{n} x_i \\ \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} z_i \end{pmatrix}, \tag{4}$$

established of substitution

$$d = -ax_C - by_C - cz_C\,, \tag{5}$$

is derived formula for the orthogonal distances δ_i

$$\delta_i = \frac{a(x_i - x_C) + b(y_i - y_C) + c(z_i - z_C)}{\sqrt{a^2 + b^2 + c^2}}\,. \tag{6}$$

Desired vector $\boldsymbol{n} = (a, b, c)^T$ minimizing the sum of squares (6)

$$\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} \frac{\left(a(x_i - x_C) + b(y_i - y_C) + c(z_i - z_C)\right)^2}{\left(\sqrt{a^2 + b^2 + c^2}\right)^2} \tag{7}$$

takes the form in matrix notation

$$\sum_{i=1}^{n}\delta_i^2=\frac{(\boldsymbol{M\,n})^T(\boldsymbol{M\,n})}{\boldsymbol{n}^T\,\boldsymbol{n}}=\frac{\boldsymbol{n}^T\,\boldsymbol{M}^T\,\boldsymbol{M\,n}}{\boldsymbol{n}^T\,\boldsymbol{n}}=\frac{\boldsymbol{n}^T\,\boldsymbol{A\,n}}{\boldsymbol{n}^T\,\boldsymbol{n}}, \quad (8)$$

wherein (9) – *frame on bottom of this page*.

Vector *n*minimizes the relationship (8) can be determined by Rayleigh quotient (Strang, 2006)

$$R(\boldsymbol{n}_1)=\frac{\boldsymbol{n}_1^T\,\boldsymbol{A\,n}_1}{\boldsymbol{n}_1^T\,\boldsymbol{n}_1}=\lambda_1, \quad (10)$$

which represents the minimum eigenvalue λ_1 of matrix $\boldsymbol{A}$. Unknown components of the normal vector of the plane $\boldsymbol{n}=(a,b,c)^T$ are made up of elements of the eigenvector corresponding to the minimum eigenvalue λ_1. The fourth parameter of the general form of plane equation (1) shall be determined by the relationship

$$d=-\frac{1}{n}\sum_{i=1}^{n}(ax_i+by_i+cz_i). \quad (11)$$

Robust M–estimation of orthogonal distance plane fitting

The algorithm described in the previous section gives good results of estimated plane parameters in case no outliers occur in measured data sets. However, if the measured 3D coordinates contains blunders, their impact is necessary to eliminate, or at least as much to suppress. In the first of these options is gross error excluded from measured data according to statistical tests that assume only one gross error in the measured data seto, it is better to suppress the distorting effect of one or more outlying points by the suitable robust methods which are currently still developing and finding some new applications in the processing of experimental data.

Let's use the linear mixed model (12) to get the general form of the plane equation (Leick, 1980)

$$\boldsymbol{F}(\widehat{\boldsymbol{L}},\widehat{\boldsymbol{X}})=\boldsymbol{0}, \quad (12a)$$

$$\boldsymbol{G}(\widehat{\boldsymbol{X}})=\boldsymbol{0}, \quad (12b)$$

where

$$F(\widehat{x}_i,\widehat{y}_i,\widehat{z}_i,\widehat{a},\widehat{b},\widehat{c},\widehat{d})=ax+by+cz+d=0,$$

$$\widehat{\boldsymbol{X}}=\boldsymbol{X}_0+\Delta\widehat{\boldsymbol{X}};\quad \begin{pmatrix}\widehat{a}\\ \widehat{b}\\ \widehat{c}\\ \widehat{d}\end{pmatrix}=\begin{pmatrix}a_0\\ b_0\\ c_0\\ d_0\end{pmatrix}+\begin{pmatrix}\Delta\widehat{a}\\ \Delta\widehat{b}\\ \Delta\widehat{c}\\ \Delta\widehat{d}\end{pmatrix}$$

is vector of adjusted unknown parameters of general form of plane equation (1). Vector $\boldsymbol{X}_0=(a_0\ b_0\ c_0\ d_0)^T$ is the vector of approximate values. $\boldsymbol{L}$ is vector of observations, $\Delta\boldsymbol{L}=\boldsymbol{L_0}-\boldsymbol{L}$, $\boldsymbol{L}_0$ is vector of approximate values computed from the approximate values $\boldsymbol{X}_0$ and $\boldsymbol{V}=\widehat{\boldsymbol{L}}-\boldsymbol{L}=(v_{x_1},v_{y_1},v_{z_1},v_{x_2}\ \cdots)^T$ is vector of residuals.

With regard to (12b), it is appropriate to rewrite the condition (2), on the shape

$$\widehat{a}^2+\widehat{b}^2+\widehat{c}^2-1=0, \quad (13)$$

which the linearized form

$$\left\{\frac{\partial G}{\partial\widehat{X}}\right\}_{\boldsymbol{X}_0}\Delta\widehat{\boldsymbol{X}}+\boldsymbol{G}(X_0)=\boldsymbol{0}, \quad (14)$$

has in matrix notation shape

$$\boldsymbol{C}\Delta\widehat{\boldsymbol{X}}+\boldsymbol{W}_C=\boldsymbol{0}. \quad (15)$$

The linear form of the model (12) is then expressed as a system of equations

$$\boldsymbol{BV}+\boldsymbol{A}\Delta\widehat{\boldsymbol{X}}+\boldsymbol{W}=\boldsymbol{0}, \quad (16a)$$

$$\boldsymbol{C}\,\Delta\widehat{\boldsymbol{X}}+\boldsymbol{W}_C=\boldsymbol{0}, \quad (16b)$$

where $\boldsymbol{A}=\left\{\frac{\partial F(\widehat{\boldsymbol{L}},\widehat{X})}{\partial\widehat{\boldsymbol{X}}}\right\}_{X_0}$ and $\boldsymbol{B}=\left\{\frac{\partial F(\widehat{L},\widehat{X})}{\partial\widehat{\boldsymbol{L}}}\right\}_{\boldsymbol{X}_0,\boldsymbol{L}}$.

The minimizing a function of the least squares method (LS) $\boldsymbol{V}^T\boldsymbol{PV}=min.$, taking into account (16a) and (16b), takes a shape (17) – *frame on bottom of this page* (Leick, 1980) where $\boldsymbol{K}_L$, $\boldsymbol{K}_C$ are Lagrange multipliers, $\boldsymbol{P}$ is a weight matrix. Taking the partial derivatives by $\boldsymbol{V}$, $\Delta\widehat{\boldsymbol{X}}$, $\boldsymbol{K}_L$, a $\boldsymbol{K}_C$ and equating them to zero, the minimizing function (17) leads to equations

$$\boldsymbol{A}=\boldsymbol{M}^T\,\boldsymbol{M}=\begin{pmatrix}\sum_{i=1}^{n}(x_i-x_C)^2 & \sum_{i=1}^{n}(x_i-x_C)(z_i-z_C) & \sum_{i=1}^{n}(x_i-x_C)(z_i-z_C)\\ \sum_{i=1}^{n}(x_i-x_C)(y_i-x_C) & \sum_{i=1}^{n}(y_i-y_C)^2 & \sum_{i=1}^{n}(y_i-y_C)(z_i-z_C)\\ \sum_{i=1}^{n}(x_i-x_C)(z_i-z_C) & \sum_{i=1}^{n}(y_i-y_C)(z_i-z_C) & \sum_{i=1}^{n}(z_i-z_C)^2\end{pmatrix} \quad (9)$$

$$\Phi(\boldsymbol{V},\Delta\widehat{\boldsymbol{X}},\boldsymbol{K}_L,\boldsymbol{K}_C)=\boldsymbol{V}^T\boldsymbol{PV}-2\boldsymbol{K}_L^T(\boldsymbol{BV}+\boldsymbol{A}\Delta\widehat{\boldsymbol{X}}+\boldsymbol{W})-2\boldsymbol{K}_C^T(\boldsymbol{C}\,\Delta\widehat{\boldsymbol{X}}+\boldsymbol{W}_C)=min., \quad (17)$$

$$\begin{pmatrix} -\underset{n,n}{\boldsymbol{P}} & \underset{n,r}{\boldsymbol{B}^T} & \underset{n,u}{\mathbf{0}} & \underset{n,s}{\mathbf{0}} \\ \underset{r,n}{\boldsymbol{B}} & \underset{r,r}{\mathbf{0}} & \underset{r,u}{\boldsymbol{A}} & \underset{r,s}{\mathbf{0}} \\ \underset{u,n}{\mathbf{0}} & \underset{u,r}{\boldsymbol{A}^T} & \underset{u,u}{\mathbf{0}} & \underset{u,s}{\boldsymbol{C}^T} \\ \underset{s,n}{\mathbf{0}} & \underset{s,r}{\mathbf{0}} & \underset{s,u}{\boldsymbol{C}} & \underset{s,s}{\mathbf{0}} \end{pmatrix} \begin{pmatrix} \underset{n,1}{\mathbf{V}} \\ \underset{r,1}{\mathbf{K}_L} \\ \underset{u,1}{\Delta\widehat{\mathbf{X}}} \\ \underset{s,1}{\mathbf{K}_C} \end{pmatrix} = \begin{pmatrix} \underset{n,1}{\mathbf{0}} \\ \underset{r,1}{-\boldsymbol{W}} \\ \underset{\boldsymbol{u,1}}{\mathbf{0}} \\ \underset{s,1}{\mathbf{W}_C} \end{pmatrix} \tag{18}$$

and after modification

$$\begin{pmatrix} \boldsymbol{A}^T\boldsymbol{M}^{-1}\boldsymbol{A} & \boldsymbol{C}^T \\ \boldsymbol{C} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \Delta\widehat{\boldsymbol{X}} \\ \boldsymbol{K_C} \end{pmatrix} = \begin{pmatrix} -\boldsymbol{A}^T\boldsymbol{M}^{-1}\boldsymbol{W} \\ -\boldsymbol{W_C} \end{pmatrix}. \tag{19}$$

The number of unknown parameters u, number of equations r indicated by (12a), number of conditions $s = 1$ given by relation (13) and number of observations $n = 3\times$ {*number of 3D points*} must satisfy the conditions:

$$\begin{array}{ccccc} n+u & > & r+s & > & u \\ n+s-u & > & r & > & u-s \end{array}. \tag{20}$$

Taking into account the degree of freedom $r+s-u$, the estimate of the a posteriori variance of unit weight is given

$$s_0^2 = \frac{\boldsymbol{V}^T\boldsymbol{P}\,\boldsymbol{V}}{r+s\text{-}u} \tag{21}$$

and variance-covariance matrix of the unknown parameters (22) – *frame below*, where $\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{P}^{-1}\boldsymbol{B}^T$. There are often occur situations in the processing of the measurement that one or more blunders penetrated into vector of observations (Labant et al., 2011), as a result of inaccurate determination of their weights (wrongly-set stochastic model), or incomplete formation of geometrical and physical relationships between measurements and unknown parameters (twrongly-set functional model) which leads to the fact that observations do not have a normal distribution $N(\mu, \sigma^2)$ with mean μ and variance σ^2. Therefore, robust M-estimates, which are the generalized form of the maximum likelihood estimations, introducted by Huber (1964) has developed particularly by courtesy the development and availability of computer technology.

In general, these estimates minimize not only the expression $\boldsymbol{V}^T\boldsymbol{P}\,\boldsymbol{V}$ but also the more general term $f((\boldsymbol{V}, \boldsymbol{L}, \boldsymbol{X}), \boldsymbol{P}) \rightarrow min$.

The diagonal elements p_i of the weighting matrix

$$\underset{n,n}{\boldsymbol{P}} = \begin{pmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & p_n \end{pmatrix} = \sigma_0^2 \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\sigma_n^2} \end{pmatrix}, \tag{23}$$

are changed in each subsequent iteration $(k+1)$ according to the rule (Therefore appellation iteratively re-weighting the LS is also used for these methods.)

$$\underset{n,n}{\boldsymbol{\Sigma}_X} = s_0^2\left[\left(\boldsymbol{A}^T\ \boldsymbol{M}^{-1}\ \boldsymbol{A}\right)^{-1} - \left(\boldsymbol{A}^T\ \boldsymbol{M}^{-1}\ \boldsymbol{A}\right)^{-1}\boldsymbol{C}^T\left\{\boldsymbol{C}\left(\boldsymbol{A}^T\ \boldsymbol{M}^{-1}\ \boldsymbol{A}\right)^{-1}\boldsymbol{C}^T\right\}^{-1}\boldsymbol{C}\left(\boldsymbol{A}^T\ \boldsymbol{M}^{-1}\ \boldsymbol{A}\right)^{-1}\right], \tag{22}$$

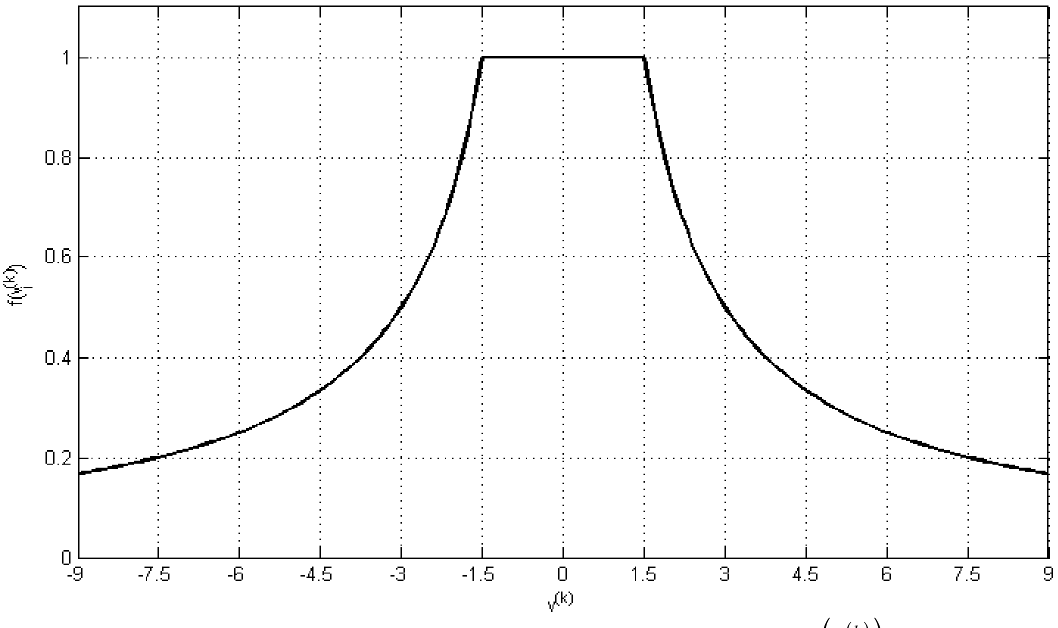


Fig. 1. Graphical representation of the Huber's weight function $f\left(v_i^{(k)}\right)$ (25)

Rys. 1. Graficzne przedstawienie funkcji wagowej Hubera $f\left(v_i^{(k)}\right)$ (25)

$$\left(p_i^{(k+1)}\right) = p_1^{(k)} f\left(v_i^{(k)}\right), \tag{24}$$

where σ_i^2 are variances of observations and σ_0^2 is the a priori variance of unit weight.

There are a number of theoretically derived and published forms for weighting function $f(v_i)$ (eg Andrews, 1974; Hampel, 2001; Huber, 1981; Yang et al., 2002; Schaffrin, 1997; Krarup et al. 1980), from which the robust M-estimation by Huber (Huber, 1981) has been used for the purpose of robust orthogonal fitting of plane

$$f\left(v_i^{(k)}\right) = \begin{cases} 1 & if \quad v_i^{(k)} \le c \\ \dfrac{c}{\left|v_i^{(k)}\right|} & if \quad v_i^{(k)} > c \end{cases}, \tag{25}$$

where k is the number of iterations and

$$c = 1.5\sigma_{L_i} = 1.5\frac{\sigma_0}{\sqrt{p_i}}.$$

Numerical example and comparisons

Positive features of a robust M-estimation by Huber in relation to the least squares method are shown by the example of the plane, expressed in intercept form (Bartsch, 2000)

$$\frac{x}{P_x} + \frac{y}{Q_y} + \frac{z}{R_z} = 1, \tag{26}$$

where $P_x = 8\ cm$, $Q_y = 9\ cm$, $R_z = 12\ cm$ are the intersection points of the plane with the axis of coordinates. The plane in the shape of (26) corresponds to the general form of the equation (1) with coefficients $a = -\frac{d}{P_x} = 0.6690$, $b = -\frac{d}{Q_y} = 0.5946$, $c = -\frac{d}{R_z} = 0.4460$ and $d = -\frac{1}{\sqrt{\left(\frac{1}{P_X}\right)^2 + \left(\frac{1}{Q_y}\right)^2 + \left(\frac{1}{R_z}\right)^2}} = -5.3517$. In this plane was chosen 16 points, therefore, $r = 16$, $n = 3 \times 16 - 48$. The observed coordinates of points have been devalued by the errors, subordinated to normal distribution with a priori standard devation $\sigma = 0.05\ cm$. In addition, gross errors have been added to the coordinates of points 2 and 12. (Chart 1, Fig. 2).

Chart 1. Comparison of measured points orthogonal distances to the estimated plane surface and their components in the coordinate axis

Tabela 1. Porównanie odległości prostopadłych mierzonych punktów do szacowanej płaskiej powierzchni i ich elementów na osi współrzędnych

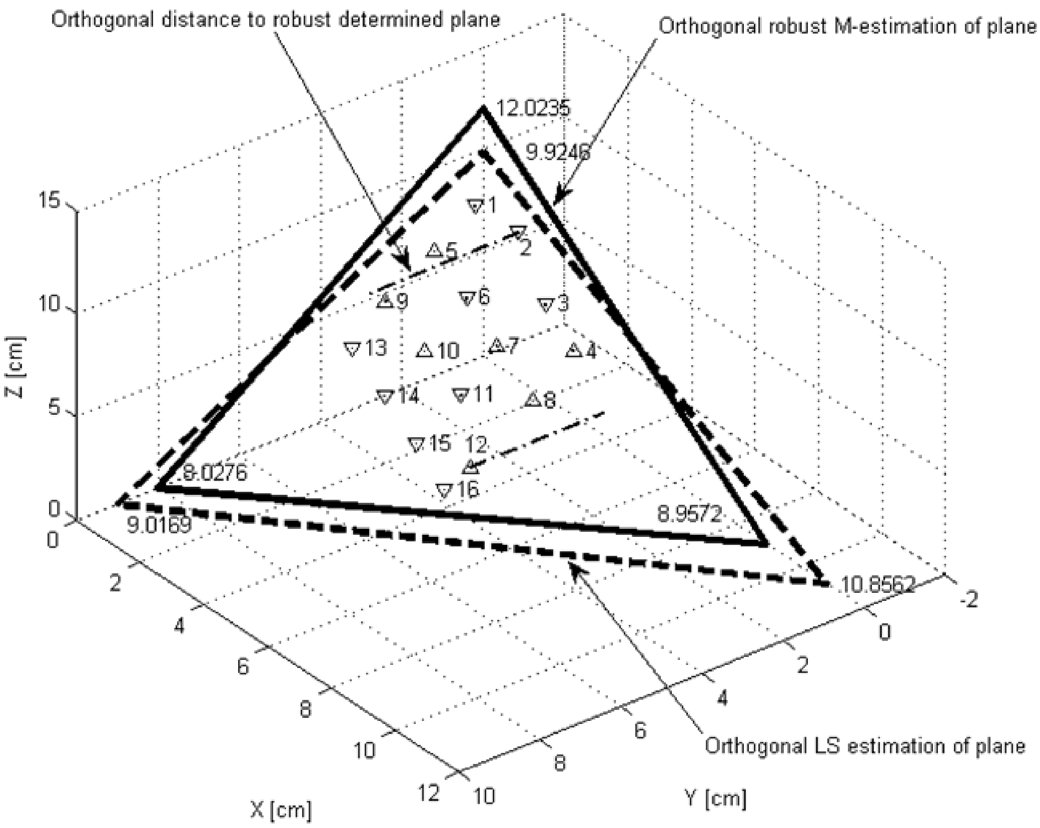
3D points				Results of LS adjustment				Results of robust M–estimation			
No.	x [cm]	y [cm]	z [cm]	δ [cm]	v_x [cm]	v_y [cm]	v_z [cm]	δ [cm]	v_x [cm]	v_y [cm]	v_z [cm]
1	1.00	1.00	9.10	0.682	-0.430	-0.357	-0.391	-0.037	0.025	0.022	0.017
2	-0.40	0.60	6.30	-2.014	1.270	1.055	1.154	-2.456	3.678	0.004	0.003
3	0.90	3.10	6.40	0.172	-0.108	-0.090	-0.098	-0.051	0.034	0.030	0.023
4	1.00	4.10	5.20	0.071	-0.045	-0.037	-0.041	0.079	-0.053	-0.047	-0.035
5	2.00	1.00	7.70	0.510	-0.322	-0.267	-0.292	0.006	-0.004	-0.004	-0.003
6	1.90	1.90	6.30	0.117	-0.074	-0.061	-0.067	-0.146	0.097	0.087	0.065
7	2.00	3.00	5.10	0.068	-0.043	-0.036	-0.039	0.044	-0.029	-0.026	-0.020
8	2.00	4.10	3.60	-0.215	0.136	0.113	0.123	0.034	-0.022	-0.020	-0.015
9	3.20	1.00	6.20	0.408	-0.257	-0.214	-0.234	0.139	-0.093	-0.083	-0.062
10	3.10	2.10	4.80	0.119	-0.075	-0.062	-0.068	0.106	-0.071	-0.063	-0.047
11	2.90	3.00	3.50	-0.281	0.177	0.147	0.161	-0.068	0.045	0.041	0.030
12	4.40	5.20	3.40	1.760	-1.110	-0.922	-1.008	2.202	-3.296	-0.005	-0.004
13	4.00	1.00	4.60	-0.004	0.003	0.002	0.003	-0.040	0.027	0.024	0.018
14	4.00	2.00	3.30	-0.225	0.142	0.118	0.129	-0.021	0.014	0.013	0.009
15	4.00	3.00	2.00	-0.446	0.282	0.234	0.256	-0.002	0.002	0.001	0.001
16	4.00	3.90	0.70	-0.720	0.454	0.377	0.412	-0.043	0.029	0.026	0.019

Captions:
x, y, z Observed coordinates of 3D points
δ Orthogonal distance to the estimated plane (3)
v_x, v_y, v_z Errors of coordinates (18)
p_x, p_y, p_z Weights of observed coordinates (23

Chart 2. Comparison of plane equations parameters in intercept and general form

Tabela 2. Porównanie parametrów równania płaszczyzny na przecięciu z osią oraz postać ogólna

Plane			Minimized function		
Equation form	Parameters	Deterministic	$\sum_{i=1}^{n} \delta_i^2$	$\boldsymbol{V}^T \boldsymbol{P} \boldsymbol{V}$	$f((\boldsymbol{V}, \boldsymbol{L}, \boldsymbol{X}), \boldsymbol{P})$
			Orthogonal LS	Orthogonal LS	Orthogonal robust by Huber
Intercept: $\frac{x}{P_x} + \frac{y}{Q_y} + \frac{z}{R_z} = 1$	P_x	8.000 000	9.016 921	9.016 921	8.027 637
	Q_y	9.000 000	10.856 243	10.856 243	8.957 215
	R_z	12.000 000	9.924 639	9.924 639	12.023 482
General: $ax + by + cz + d = 0$	a	0.668 965	0.630 529	0.630 529	0.666 817
	b	0.594 635	0.523 702	0.523 702	0.597 615
	c	0.445 976	0.572 861	0.572 861	0.445 209
	d	-5.351 718	-5.685 434	-5.685 434	-5.352 966



Captions:

Δ Observed 3D points above the plane determined by Huber's method

∇ Observed 3D points below the plane determined by Huber's method

Fig.2. Graphical projection of observed 3D points and planes determined by LSM and robust M-estimation by Huber

Rys. 2. Graficzna projekcja obserwowanych punktów 3D i płaszczyzn określonych przez LSM oraz mocną M-estymacją według Hubera

The sizes and structures of the matrices according to (18) are:

$$\underset{16,4}{\boldsymbol{A}} = \left\{ \frac{\partial F\left(\widehat{L}, \widehat{X}\right)}{\partial \widehat{X}} \right\}_{X_0} = \begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ \vdots & & & \\ x_{16} & y_{16} & z_{16} & 1 \end{pmatrix}, \qquad \underset{16,1}{\boldsymbol{W}} = \boldsymbol{F}(\boldsymbol{L}_0, \boldsymbol{X}_0) = \begin{pmatrix} a_0 x_1 + b_0 y_1 + c_0 z_1 + d_0 \\ a_0 x_2 + b_0 y_2 + c_0 z_2 + d_0 \\ \vdots \\ a_0 x_{16} + b_0 y_{16} + c_0 z_{16} + d_0 \end{pmatrix},$$

$$\underset{16,48}{\boldsymbol{B}} = \left\{ \frac{\partial F\left(\widehat{L}, \widehat{X}\right)}{\partial \widehat{X}} \right\}_{X_0, L} = \begin{pmatrix} a_0 & b_0 & c_0 & & & & & & \\ & & & a_0 & b_0 & c_0 & & & \\ & & & & & \vdots & & & \\ & & & & & & a_0 & b_0 & c \end{pmatrix},$$

$$\underset{1,4}{\boldsymbol{C}} = \left\{ \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}} \right\}_{\boldsymbol{X}_0} = \begin{pmatrix} 2a_0 & 2b_0 & 2c_0 & 0 \end{pmatrix}, \qquad \underset{1,1}{\boldsymbol{W}_{\boldsymbol{C}}} = \boldsymbol{G}(X_0) = a_0^2 + b_0^2 + c_0^2 - 1 .$$

Iterative calculation was stopped after 33 iterations when maximal difference between weights in iteration steps (k) and $(k+1)$ of each observations satisfy condition

$$\max\left(\left|p_i^{(k)} - p_i^{(k+1)}\right|\right) < 1.0e-05 ,$$

where $1.0e-05$ is chosen constant.

Conclusion

The results presented in the tables below (Chart 1, Chart 2) clearly show that the robust methods (in this case, robust M-estimation by Huber) are quite simple, yet very efficient tool to eliminate the negative influence of gross errors contained in the observed 3D coordinates on the estimates of the parameters of the plane. Degradation effect of gross errors by robust M-estimations is reflected not only on the more objective fitting of plane, but also in improve the estimation of a posteriori standard deviations s_0 (21), which compared with the method of least squares was reduced from $0.802\,cm$ to $0.088\,cm$, which corresponds better to the a priori standard deviation $\sigma = 0.05\,cm$.

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Dopasowanie ortogonalnej odległości płaszczyzny

Ortogonalne dopasowanie odległości płaskiej powierzchni na punkty 3D metodą najmniejszych kwadratów jest najlepszym rozwiązaniem w przypadku gdy błędy grube oraz błędy systematyczne nie wpływają na obserwacje. Takie sytuacje zdarzają się jednak często podczas przetwarzania danych eksperymentalnych, a metody odpornościowe są dobrą alternatywą w przypadkach tego typu. Ten problem został przedstawiony na przykładzie dopasowania ortogonalnego płaszczyzny na zestaw punktów 3D przy użyciu metody M-estymacji opracowanej przez Hubera.

Słowa kluczowe: Dane skaningowe 3D, ortogonalne dopasowanie odległości płaszczyzny, odporność, M-estymacja, wartość oddalona