Deformation Analysis of the Network Structure Measured by Global Navigation Satellite Systems with a Selection of Cofactors

Gabriel WEISS [1], Slavomír LABANT [2], Erik WEISS [3]

[1] Dr. h. c. prof. Ing., Ph.D.; Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnology, TUKE – Technical University of Košice, Park Komenského 19, 040 01 Košice, Slovak Republic; e-mail: gabriel.weiss@tuke.sk, tel.: +421 55 602 2009

[2] Doc. Ing., Ph.D.; Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnology, TUKE – Technical University of Košice, Park Komenského 19, 040 01 Košice, Slovak Republic; e-mail: slavomir.labant@tuke.sk, tel.: +421 55 602 2859

[3] Doc. Ing., Ph.D.; Institute of Geotourism, Faculty of Mining, Ecology, Process Control and Geotechnology, TUKE – Technical University of Košice, Park Komenského 19, 040 01 Košice, Slovak Republic; e-mail: Erik.Weiss@tuke.sk, tel.: +421 55 602 2332

Summary

The present paper deals with the deformation analysis of a 3D geodetic network at the Cierny Vah water transfer power station observed by Global Navigation Satellite Systems technology in the years 2004 and 2008. The aim of the work is to evaluate the influence of parameters entering the model, estimate parameters of the first and second grades of the network structures and present the results of the deformation analysis with graphic visualisation of individual processes and analyses. Four types of cofactors or weights were used to process and adjust the observations as recommended by the manufacturer of the GNSS receiver by way of substitution of a covariance matrix from the Spectrum Survey using RMS and constant 1 as an a priori nominal variance factor. The MINQUE method was also used to estimate cofactors of the observation components. The greatest weights were estimated by the application of the MINQUE method and assigned to their vectors. Based on the coordinate estimates of the determined points, a solution with cofactors using the covariance matrix proved to be the one that has the least deviations. From the viewpoint of standard deviations, the solution using the covariance matrixes from the Spectrum Survey achieved the highest degree of accuracy. Numerical results from processing showed the use of the MINQUE method as a suitable alternative to laborious input of covariance matrixes into the Spectrum Survey software environment.

Keywords: 3D local geodetic net, GNSS surveying, LMS, MINQUE, deformation analysis, errors ellipsoid

Introduction

At present, various methods are used to monitor area stability, mainly of a terrestrial nature. These are terrestrial measurements by a universal measurement station, very precise levelling, digital photogrammetry, laser scanning as well as using global navigation satellite systems (GNSS) which have significantly increased in accuracy over the last decade. During a GNSS observation, one of the conditions which must be met is a clear sky or direct visibility to the satellites. The benefits of using satellite technology are mainly in independence from direct visibility between points which provides the possibility for very accurate deformation measurement without making varying and significant alterations to the environment necessary for taking the sightings. The use of GNSS technology is suitable in areas of higher environmental protections since it is not necessary to cut through vegetation.

Selection of a coordinate system for processing

GNSS work with a WGS-84 geocentric coordinate system connected to Earth. GNSS receivers provide the results in Cartesian ellipsoid-centric coordinates related to the WGS-84. In our continent, spatial coordinates obtained when using GNSS are generally related to ETRS-89 referential system with GRS-80 referential ellipsoid. The reason for creating ETRS-89 is the movement of the Eurasian continental plate which causes a change in the coordinates of fixed points on Earth. The difference in the horizontal position between WGS-84 and ETRS-89 increases by 2.7 cm per year. The heights are the same in both systems (Leick, 1995).

For 3D GNSS network structures for explicit positioning in a particular 3D coordinate system, it is sufficient to select only one point from the set of reference points available for adjustment, which must be stable in the environment. The advantage of this connection is that the network will not be deformed by the influence of other reference points. If this point itself is incompatible during repeated measurements, the same distortion of coordinates of individual points will take place, which will not be shown in the difference in final coordinates. After observation, the data will be transferred to the appropriate program environment where the measured GPS (GNSS) vectors between observed points are processed. Processed output values are given in *.txt format or *.html and contain information about the vector, its length, sighting accuracy, period of observation, frequency of signal received, entry values of temperature and pressure for atmospheric adjustment, Cartesian and geographical coordinates of primary and ultimate vector points as well as other data. The values entering the calculation are geodetic geographic coordinates (ϕ, λ, h) determined from GNSS measurements which can be transferred into 3D Cartesian coordinates in a selected geodetic 3D system (Weiss, 1997, Seeber, 2003). From the approximate geodetic geographic coordinates (from GNSS technology), 3D Cartesian coordinates will be obtained which will be used in further processing.

Gauss-Markov estimation model for processing a deformation net

A Gauss-Markov model (GMM) is the most frequently used method for adjustment of a geodetic network according to the equation 1, where v represents the correction vector for observed values, A is the design matrix, $d\hat{C} = \hat{C} - C^\circ$ is the vector of an increase of adjusted values of determined coordinates, $dL = L - L^\circ$ is the vector of reduced observations.

The measured GNSS data of vectors, primarily processed using software, can be processed on the basis of Gauss-Markov estimation model (adjustment of observation equations) with full rank. We consider spatial Cartesian coordinates of a reference point obtained from a software solution as fixed. We seek adjusted coordinates $\hat{C}$ of network points via sighted and pre-processed values in the network. In this case, the GNSS observation vectors ΔXYZ_{ij} are lined into vector L in the order according to the equation 2 (Gašinec, Gašincová, 2005; Labant, 2008; Gašincová, et al., 2011).

Observation vector of implemented sightings L creates m – of observation vectors, i.e. $n=3m$ observation components.

$\underset{(k,1)}{C^\circ}$ – represents vector of approximate coordinates of determined points with a column structure:

$$\underset{(k,1)}{C^\circ} = \begin{pmatrix} X_i^\circ \\ Y_i^\circ \\ Z_i^\circ \end{pmatrix} \text{ where } i = 1, \ ..., b \text{ points} \tag{3}$$

$\underset{(n,1)}{L^\circ}$ – vector of approximate values of observations, which we retrospectively obtain from the vector of approximate coordinates $\underset{(k,1)}{C^0}$ can be expressed as a vector of functions $L^\circ = f(C^\circ)$.

$\underset{(n,1)}{dL}$ – vector of auxiliary measured values:

$$\underset{(n,1)}{dL} = \underset{(n,1)}{L} - \underset{(n,1)}{L^\circ}. \tag{4}$$

$\underset{(n,1)}{v}$ – correction vector, we obtain:

$$\underset{(n,1)}{v} = \underset{(n,k)}{A} \cdot \underset{(k,1)}{d\hat{C}} - \underset{(n,1)}{dL}, \tag{5}$$

where A is the design matrix – matrix of function partial derivation $\underset{(n,1)}{L^\circ} = f(C^\circ)$ according to the vector of determined parameters $\underset{(k,1)}{C^\circ}$:

$$\underset{(n,k)}{A} = \left(\frac{\partial L^\circ}{\partial C^\circ} = \frac{\partial f(C^\circ)}{\partial C^\circ} = \{-1,\ 0,\ 1\} \right)_{\hat{C} \doteq C^\circ} \tag{6}$$

$\underset{(n,n)}{Q_L}$ – diagonal matrix of vector cofactors $\underset{(n,1)}{L}$, which will have the structure represented by the equation 7, $Q_{l_{ij}}$ correspond to individual GNSS observation vectors ΔXYZ_{ij}, elements in submatrix $Q_{l_{ij}}$ on the main diagonals are cofactors of values $(\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij})$, elements outside the main diagonal are mixed cofactors.

$$\begin{aligned} v &= Ad\hat{C} - dL = A(\hat{C} - C^\circ) - (L - L^\circ), && - \textit{functional segment} \\ \Sigma_L &= s_0^2 Q_L && - \textit{stochastic segment} \end{aligned} \tag{1}$$

$$\underset{(m,1)}{L} = (\Delta XYZ_{ij}), \ where: \underset{(3,1)}{\Delta XYZ} = \begin{pmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \end{pmatrix} - \text{observation components} \tag{2}$$

$$\underset{(n,n)}{Q_L} = \begin{pmatrix} \underset{(3,3)}{Q_{l_{12}}} & & & \\ & \ddots & & \\ & & \underset{(3,3)}{Q_{l_{ij}}} & \\ & & & \ddots \end{pmatrix} \quad \begin{matrix} \text{with co-factor} \\ \text{submatices:} \end{matrix} \quad \underset{(3,3)}{Q_{l_{ij}}} = \begin{pmatrix} q_{\Delta X \Delta X} & q_{\Delta X \Delta Y} & q_{\Delta X \Delta Z} \\ q_{\Delta Y \Delta X} & q_{\Delta Y \Delta Y} & q_{\Delta Y \Delta Z} \\ q_{\Delta Z \Delta X} & q_{\Delta Z \Delta Y} & q_{\Delta Z \Delta Z} \end{pmatrix}_{ij} \tag{7}$$

Apart from measured observations, entry data for such a constructed model is L_{ij} and approximately values of coordinates $C_i°$ as well as selection of cofactors q_{ij}. From the various types, the following four cofactors were used for processing (Caspary, 1987; Sabova, Jakub, 2007; Pukanská, Weiss, 2007) (Tab. 2):

1. ${}^{(1)}q_{l\,ij} = 5\ mm\ +\ 1 \cdot D\ ppm$ recommended by the GNSS receiver producer for a static method,
2. ${}^{(2)}q_{l\,ij}$ from covariance matrices $\Sigma_{l\,ij}$, obtained from processing observations in a program environment for individual GNSS vectors with empiric a posteriori nominal variance $s_0^2 = 1$,
3. ${}^{(3)}q_{l\,ij}$ from covariance matrixes $\Sigma_{l\,ij}$, with empiric a posteriori nominal variance $s_0^2 = RMS$ (root mean square) also obtained from the same pre-processed values,
4. ${}^{(4)}q_{l\,ij}$ using the MINQUE method for estimation of cofactors (Skořepa, Dušek, 1998).

The values of parameters related to coordinates of determined network points also depend upon selection of cofactors of the measured values. $\underset{(k,1)}{\hat{C}}$ – vector of estimations of observed point coordinates is determined (Gašinec, Gašincová, 2005):

$$\underset{(k,1)}{\hat{C}} = \underset{(k,1)}{C°} + \underset{(k,1)}{d\hat{C}}, \tag{8}$$

where $d\hat{C}$ is the vector of estimates of auxiliaries of determined coordinates, most frequently in *mm*:

$$d\hat{C} = \left(A^T Q_L^{-1} A\right)^{-1} A^T Q_L^{-1}(L - L°) = N^{-1} A^T Q_L^{-1} dL, \tag{9}$$

where A and dL are the same for all used processing methods, differences in $d\hat{C}$ depend upon the used type of cofactors q_{lij}. Standard deviations of estimates of determined point coordinates also depend on cofactors $s_{\hat{C}}$ obtained from the main diagonal of covariance matrix:

$$\Sigma_{\hat{C}} = s_0^2 . Q_{\hat{C}}, \tag{10}$$

where the cofactor matrix is $Q_{\hat{C}} = \left(A^T . Q_L^{-1} . A\right)^{-1}$ and nominal a priori variance factor is $s_0^2 = \dfrac{v^T . Q_L^{-1} . v}{n-k}$.

One of the methods used for estimation of the elements of matrix Q_L is the MINQUE method – Minimum Norm Quadratic Unbiased Estimation (Skořepa et al., 1998; Gašinec et al., 2005). The solution of the task is based on the functional part of the Gauss-Markov estimation model $v = A \cdot d\hat{C} - dL$, for the vector of residuals. Combination of various geodetic devices and measuring methods used to obtain a vector of the measured values in the terrain are described by the stochastic part of the model as follows:

$$C_l = \vartheta_X^0 V_X + \vartheta_Y^0 V_Y + \vartheta_Z^0 V_Z, \tag{11}$$

where coefficients of the linear combination ϑ_i^0 are a priori variance components, V_i are corresponding positive semi definite matrices of order n. Impartial invariant quadratic estimate (MINQUE) with a minimum norm of variance components $(\vartheta_X^0, \vartheta_Y^0, \vartheta_Z^0)^T$ for the estimation of parameters of order 2 is given by the relation 12.

Matrix M is calculated from the relation:

$$M = C_l^{-1} - C_l^{-1} A (A^T C_l^{-1} A)^{-1} A^T C_l^{-1}. \tag{13}$$

The estimated coefficients of the linear combination $(\hat{\vartheta}_X^0,\ \hat{\vartheta}_Y^0,\ \hat{\vartheta}_Z^0)^T$ are used again in the relation (8). The relation (9) then shows the improved values. The iteration cycle is completed upon fulfillment of the condition:

$$\left|\hat{\vartheta}_i - \vartheta_i\right| \le \delta, \qquad i = 1,\ 2,\ ...,\ n, \tag{14}$$

where δ is a preselected constant from the set of real numbers by which the required degree of accuracy of the estimated parameters is defined.

Assessment of changes in the observed area in each inter-era t_i and t_{i+1} is carried out on the basis of changes in coordinates of observed points in the given area. Differences or changes in coordinates of one point within two eras of measurement from coordinates adjusted by GMM was calculated using the least square method:

$$^{t_i,t_{i+1}}\Delta\hat{C} = {}^{t_{i+1}}\hat{C} - {}^{t_i}\hat{C} = \begin{pmatrix} ^{t_i,t_{i+1}}\Delta\hat{X} \\ ^{t_i,t_{i+1}}\Delta\hat{Y} \\ ^{t_i,t_{i+1}}\Delta\hat{Z} \end{pmatrix} = \begin{pmatrix} ^{t_{i+1}}\hat{X} - {}^{t_i}\hat{X} \\ ^{t_{i+1}}\hat{Y} - {}^{t_i}\hat{Y} \\ ^{t_{i+1}}\hat{Z} - {}^{t_i}\hat{Z} \end{pmatrix} \tag{15}$$

Indicator in a change of 3D position of a point as a spatial shift is determined using the equation:

$$\begin{pmatrix} tr(MV_X MV_X) & \cdots & tr(MV_X MV_Z) \\ \vdots & \ddots & \vdots \\ tr(MV_Z MV_X) & \cdots & tr(MV_Z MV_Z) \end{pmatrix} \begin{pmatrix} \hat{\vartheta}_X^0 \\ \hat{\vartheta}_Y^0 \\ \hat{\vartheta}_Z^0 \end{pmatrix} = \begin{pmatrix} dL^T MV_X MdL^T \\ \vdots \\ dL^T MV_Z MdL^T \end{pmatrix} \tag{12}$$

$$ {}^{t_i,t_{i+1}}\Delta\hat{X}\hat{Y}\hat{Z} = \sqrt{{}^{t_i,t_{i+1}}\Delta\hat{X}^2 + {}^{t_i,t_{i+1}}\Delta\hat{Y}^2 + {}^{t_i,t_{i+1}}\Delta\hat{Z}^2} \quad (16) $$

On the basis of numeric determination of the size and orientation of indicators on individual points, it is possible to evaluate whether in the observed era t_i and t_{i+1} the indicator signalises the positional or spatial constancy of a point or signalises the non-identity of the point caused by the influence of deformation forces in the given area. Geometric structures for identical (unchanged) locations of points should be stochastically identical – congruent. The fact as to whether the calculated indicators indicate a real point movement between eras or they were created by an accumulation of measurement errors has to be verified using statistic tests which evaluate the coordinate identity on the basis of certain probability of normal division.

Observation and pre-processing of a deformation network

A sighting was carried out in two independent eras, in t_1 (April 2004 – 2 days) and in t_2 (July 2008 – 1 day) on a deformation network of seven points which is shown on Fig.1. The deformation network is situated in the location of the upper reservoir of the Cierny Vah water transfer power station (WTPS) in the area of the Low Tatras National Park. Observations were carried out using the static method on the principle of relative determination of a position and primary evaluation of results was done in a post-processing regime, in the Spectrum Survey (SS) software environment. Sokkia Stratus GPS receivers with suitable properties for local deformation surveys with vectors up a distance of 10–12 km were used to receive GNSS signals in the area of interest. The producer's declared accuracy for the static method on the ground surface in a horizontal direction is $5\,mm + 1 \cdot D\ ppm$ and in a vertical direction is $10\,mm + 1 \cdot D\ ppm$ (Havasi, Györffy, 2007; Sabová, Pukanská, 2007).

In April 2004, two single frequency GPS receivers were used for observation and four single frequency GPS receivers were used in July 2008. In both eras, measurement using the static method took place gradually above all network points: 5001 to 5007. Point 5001 was selected as the reference network point since it was stabilised in the largest and most level terrain. Sighting of the GPS network was implemented via 11 vectors (Fig.1) of the 3D network in the WTPS Cierny Vah location. Signal reception on individual network points was 1 hour and 7 hours for reference point 5001. During measurement, the number of observed satellites fluctuated from 4 to 10. The measured data was primarily processed in the SS software environment. Output was geographical coordinates φ, λ and heights h of individual network points.

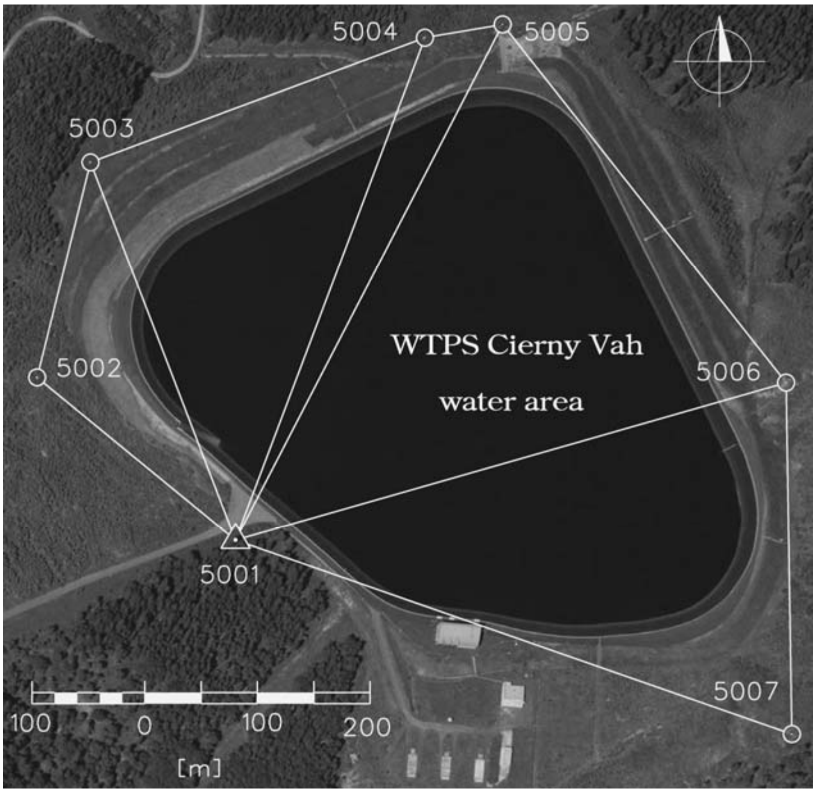


Fig. 1. Structure of 3D network in the location WTPS Cierny Vah

Rys. 1. Struktura sieci 3D w lokalizacji WTPs Cierny Vah

Comparison of separate and bivariant adjustment of a deformation net

By applying the above stated equations, we will obtain approximate measured values $L°$ and reduced observations dL which have the same values for all processing methods for eras 2004 and 2008. Various types of cofactors influence corrections of measured values v, adjusted values $\hat{L}$, standard deviations of adjusted values $s_{\hat{L}}$ characterising the accuracy of estimates in the direction of individual axes. When ascertaining the differences between separate and bivariant processing using cofactors recommended by the producer, i.e. $5\ mm + 1 \cdot D\ ppm$, differences between individual sightings were not discovered in values $\hat{L}$, v and $s_{\hat{L}}$. Therefore, when using these types of cofactors, it was proved that the number of eras has no influence upon the results of observation processing.

The MINQUE method showed the same values $\hat{L}$ and v as the use of $5\ mm + 1 \cdot D\ ppm$; however, the difference was between determination of standard deviations of the observed values $s_{\hat{L}}$. During separated adjustment, measurement carried out in 2004 showed greater accuracy than in 2008. Since during bivariant processing all measurements are processed together in one model, measurements with less accuracy influence measurements with greater accuracy which is reflected in the increase in individual deviation values. Taking into account that both eras were measured with approximately the same accuracy level, these differences are not great.

If the covariant matrixes are substituted into the cofactor matrix and $s_0^2 = 1$, $s_0^2 = RMS$ are substituted, a separate cofactor submatrix of the measured observation obtained from preprocessing observations in the SS software environment is substituted for each observed vector. Correlation between values in one observation is not ignored. For this reason, varying accuracy values occur when determining the adjusted values $\hat{L}$. Their difference between separate and bivariant adjustment fluctuates around ±0,3 mm. Adjusted values $\hat{L}$ together with corrections v do not differ between processing methods.

From the viewpoint of the number of processed eras, significant differences between processing results were not found; therefore, it is up to the person resolving the issue as to which of these methods is selected, under the condition that both eras are processed with approximately the same level of accuracy. Tab. 1, parameters estimated via bivariant processing of observations with various cofactors are shown. Separate processing more clearly illustrates the various measurement accuracies. When using software solutions for observations, the advantage is mainly simpler inclusion of other eras (e.g. MatLab, MathCad).

Comparison of accuracies in accordance with used cofactors

Observation carried out with greater accuracy should be given more weight and should receive a lesser part of overall corrections. Primary software processing assigns observations (Spectrum Survey, Leica Geo Office) weight depending upon the number of satellites from which it receives data, depending upon the constellation of satellites, the frequency of signal reception as well as other factors. Overall, from the viewpoint of standard deviations, the greatest accuracy is achieved by allocating covariance matrixes using $s_0^2 = RMS$. The average achieved value in sightings is $s_{\hat{L}} = 3{,}96\ mm$. The application method of $s_0^2 = 1$ into the model is approaching the deviation values where average accuracy achieved is $\bar{s}_{\hat{L}} = 4{,}00\ mm$.

Both methods separately evaluate individual vectors in the direction of axes of the selected coordinate system. When using cofactors according to the apparatus producer's recommendations, all measurements are given weights in accordance with the equation $5\ mm + 1 \cdot D\ ppm$. The accuracy of determination of observation vectors depends upon their length; therefore, the greater the distance between observed points, the lower accuracy is estimated. By numerical calculation of cofactors, the average accuracy of observed values was achieved $\bar{s}_{\hat{L}} = 4{,}57\ mm$.

The MINQUE method is the mathematical method which serves for determination of the best estimates for surveyed values. The average value $\bar{s}_{\hat{L}}$ by the MINQUE method is $\bar{s}_{\hat{L}} = 4{,}46\ mm$. Via its application, the greatest weight was estimated and assigned to vectors in the direction of axis Y, then X and the lowest in the direction of axis Z (Tab. 1).

Dependence of coordinate estimates of determined points upon selection of cofactors

Since the network is adjusted using reference point 5001, this reference point is not included in the adjustment and the values of standard deviations of coordinate estimates of this reference point are $s_{\hat{X}}^2 = s_{\hat{Y}}^2 = s_{\hat{Z}}^2 = 0$. Standard deviations of coordinate estimates of determined points $s_{\hat{C}}$ from processing will therefore be non-zero values.

Table 1. Observations of bivariant adjustment of a network in eras 04/08;
q: 1.) 5mm+1·ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE

Tabela 1. Wyniki dopasowania bivariantowego sieci w punktach 04/08;
q: 1.) 5mm+1·ppm, 2.) s02=1, 3.) s02=RSM, 4.) MINQUE

Era	o. No.	vector		***L***	***L°***	***dL***	***q_L***				$\hat{L}$ [m]		***v*** [mm]				$s_{\hat{L}}$ [mm]			
				[m]	[m]	[mm]	1	2	3	4	1 = 4	2 = 3	1	2	3	4	1	2	3	4
2004	1	5001 5002	ΔX	-38,650	-38,650	0	25,39	54,29	6,29	34,30	-38,648	-38,647	1,69	2,66	2,66	1,72	4,79	4,16	4,35	4,60
	2		ΔY	-210,811	-210,811	0	27,15	25,27	2,93	18,84	-210,812	-210,811	-0,51	-0,21	-0,21	-0,50	4,86	3,33	3,48	3,41
	3		ΔZ	78,128	78,128	0	25,79	51,58	5,97	63,32	78,130	78,132	2,28	3,52	3,52	2,31	4,80	4,66	4,87	6,26
	4	5001 5003	ΔX	-205,622	-205,622	0	27,10	24,65	2,85	34,30	-205,625	-205,625	-2,54	-2,69	-2,69	-2,57	4,26	3,36	3,52	4,02
	5		ΔY	-234,015	-234,015	0	27,40	21,22	2,46	18,84	-234,012	-234,011	3,01	3,93	3,93	3,00	4,27	2,83	2,96	2,98
	6		ΔZ	183,523	183,523	0	26,87	27,85	3,22	63,32	183,518	183,518	-5,42	-5,42	-5,42	-5,39	4,24	3,92	4,10	5,47
	7	5001 5004	ΔX	-379,841	-379,841	0	28,94	16,77	1,94	34,30	-379,836	-379,837	4,73	3,79	3,79	4,58	4,22	2,95	3,08	3,94
	8		ΔY	10,425	10,425	0	25,10	11,68	1,35	18,84	10,426	10,426	0,52	0,72	0,72	0,50	4,09	2,43	2,54	2,92
	9		ΔZ	287,820	287,820	0	27,96	44,65	5,17	63,32	287,827	287,826	6,62	5,98	5,98	6,53	4,17	4,12	4,31	5,35
	10	5001 5005	ΔX	-411,414	-411,414	0	29,28	63,77	7,38	34,30	-411,417	-411,420	-2,82	-5,68	-5,68	-2,70	4,23	3,33	3,48	3,94
	11		ΔY	71,412	71,412	0	25,72	43,38	5,02	18,84	71,411	71,410	-0,61	-1,73	-1,73	-0,50	4,13	2,67	2,79	2,92
	12		ΔZ	302,121	302,121	0	28,11	82,55	9,56	63,32	302,111	302,108	-10,29	-13,34	-13,34	-10,03	4,19	4,24	4,43	5,35
	13	5001 5006	ΔX	-285,825	-285,825	0	27,94	40,66	4,71	34,30	-285,827	-285,830	-1,77	-4,52	-4,52	-1,68	4,30	3,41	3,57	4,02
	14		ΔY	405,734	405,734	0	29,22	13,61	1,58	18,84	405,730	405,728	-4,09	-6,05	-6,05	-4,00	4,38	2,50	2,61	2,98
	15		ΔZ	110,219	110,219	0	26,11	35,75	4,14	63,32	110,231	110,233	12,17	13,77	13,77	12,39	4,23	3,90	4,08	5,47
	16	5001 5007	ΔX	-74,740	-74,740	0	25,75	21,26	2,46	34,30	-74,739	-74,741	0,60	-1,33	-1,33	0,66	4,83	3,53	3,69	4,60
	17		ΔY	508,856	508,856	0	30,35	17,27	2,00	18,84	508,858	508,857	1,57	0,70	0,70	1,50	5,06	2,90	3,03	3,41
	18		ΔZ	-95,338	-95,338	0	25,96	34,63	4,01	63,32	-95,344	-95,345	-5,79	-7,04	-7,04	-5,81	4,82	4,33	4,53	6,26
	19	5002 5003	ΔX	-166,978	-166,972	-6	26,70	10,39	1,20	34,30	-166,976	-166,977	1,77	0,65	0,65	1,72	4,84	3,11	3,25	4,60
	20		ΔY	-23,200	-23,204	4	25,23	8,19	0,95	18,84	-23,200	-23,200	-0,48	0,14	0,14	-0,50	4,79	2,65	2,78	3,41
	21		ΔZ	105,385	105,395	-10	26,07	15,68	1,82	63,32	105,387	105,386	2,30	1,06	1,06	2,31	4,81	3,69	3,86	6,26
	22	5003 5004	ΔX	-174,211	-174,219	8	26,77	15,17	1,76	34,30	-174,212	-174,213	-0,73	-1,52	-1,52	-0,85	4,66	3,25	3,40	4,39
	23		ΔY	244,435	244,440	-5	27,50	10,52	1,22	18,84	244,438	244,437	2,51	1,79	1,79	2,50	4,66	2,73	2,86	3,26
	24		ΔZ	104,312	104,297	15	26,05	28,45	3,30	63,32	104,309	104,308	-2,96	-3,60	-3,60	-3,08	4,61	4,30	4,49	5,97
	25	5004 5005	ΔX	-31,584	-31,573	-11	25,32	10,37	1,20	34,30	-31,581	-31,582	3,45	1,52	1,52	3,72	4,59	2,89	3,02	4,37
	26		ΔY	60,983	60,987	-4	25,61	8,25	0,96	18,84	60,986	60,985	2,87	1,55	1,55	3,00	4,54	2,48	2,60	3,24
	27		ΔZ	14,281	14,301	-20	25,14	28,74	3,33	63,32	14,284	14,282	3,10	0,68	0,68	3,44	4,56	4,38	4,58	5,93
	28	5005 5006	ΔX	125,589	125,589	0	26,27	14,43	1,67	34,30	125,590	125,590	1,05	1,16	1,16	1,02	4,66	3,28	3,43	4,39
	29		ΔY	334,316	334,322	-6	28,46	7,73	0,90	18,84	334,319	334,318	2,52	1,68	1,68	2,50	4,75	2,44	2,56	3,26
	30		ΔZ	-191,873	-191,902	29	26,96	17,89	2,07	63,32	-191,880	-191,875	-6,55	-1,89	-1,89	-6,58	4,65	3,75	3,92	5,97
	31	5006 5007	ΔX	211,088	211,085	3	27,16	9,08	1,05	34,30	211,087	211,088	-0,63	0,19	0,19	-0,66	4,88	2,80	2,93	4,60
	32		ΔY	103,129	103,122	7	26,04	6,52	0,76	18,84	103,128	103,129	-1,34	-0,25	-0,25	-1,50	4,92	2,36	2,46	3,41
	33		ΔZ	-205,581	-205,557	-24	27,10	15,50	1,80	63,32	-205,575	-205,578	6,04	3,19	3,19	5,81	4,86	3,59	3,76	6,26
2008	34	5001 5002	ΔX	-38,645	-38,645	0	25,39	44,72	4,24	34,30	-38,647	-38,648	-1,73	-3,27	-3,27	-1,73	4,79	5,30	5,01	4,60
	35		ΔY	-210,804	-210,804	0	27,15	27,73	2,63	18,84	-210,808	-210,809	-4,44	-5,14	-5,14	-4,31	4,86	4,02	3,81	3,41
	36		ΔZ	78,134	78,134	0	25,79	66,80	6,33	63,32	78,137	78,138	2,89	3,93	3,93	2,95	4,80	6,40	6,05	6,26
	37	5001 5003	ΔX	-205,632	-205,632	0	27,10	75,89	7,20	34,30	-205,633	-205,634	-0,54	-2,00	-2,00	-0,46	4,26	4,83	4,57	4,02
	38		ΔY	-234,013	-234,013	0	27,40	44,87	4,26	18,84	-234,010	-234,009	3,43	4,07	4,07	3,39	4,27	3,65	3,45	2,98
	39		ΔZ	183,516	183,516	0	26,87	128,56	12,19	63,32	183,518	183,518	1,80	2,26	2,26	1,90	4,24	5,98	5,66	5,47
	40	5001 5004	ΔX	-379,835	-379,835	0	28,94	50,35	4,78	34,30	-379,843	-379,843	-7,89	-8,32	-8,32	-7,65	4,22	4,68	4,43	3,94
	41		ΔY	10,424	10,424	0	25,10	24,61	2,33	18,84	10,429	10,430	5,37	5,80	5,80	5,47	4,09	3,46	3,27	2,92
	42		ΔZ	287,840	287,840	0	27,96	104,64	9,92	63,32	287,830	287,828	-9,53	-12,08	-12,08	-9,24	4,17	5,94	5,62	5,35
	43	5001 5005	ΔX	-411,424	-411,424	0	29,28	49,62	4,71	34,30	-411,413	-411,414	10,98	10,41	10,41	10,52	4,23	4,74	4,49	3,94
	44		ΔY	71,432	71,432	0	25,72	29,68	2,81	18,84	71,430	71,429	-2,14	-2,64	-2,64	-1,97	4,13	3,63	3,44	2,92
	45		ΔZ	302,125	302,125	0	28,11	91,61	8,69	63,32	302,127	302,124	2,40	-1,43	-1,43	2,37	4,19	5,79	5,48	5,35
	46	5001 5006	ΔX	-285,836	-285,836	0	27,94	55,56	5,27	34,30	-285,838	-285,838	-1,65	-1,93	-1,93	-1,79	4,30	4,44	4,20	4,02
	47		ΔY	405,728	405,728	0	29,22	32,07	3,04	18,84	405,723	405,723	-4,51	-5,46	-5,46	-4,39	4,38	3,43	3,25	2,98
	48		ΔZ	110,230	110,230	0	26,11	88,32	8,38	63,32	110,230	110,228	0,35	-1,69	-1,69	0,35	4,23	5,62	5,32	5,47
	49	5001 5007	ΔX	-74,748	-74,748	0	25,75	30,02	2,85	34,30	-74,747	-74,747	1,15	1,48	1,48	1,10	4,83	4,40	4,16	4,60
	50		ΔY	508,852	508,852	0	30,35	19,56	1,85	18,84	508,854	508,854	1,88	1,76	1,76	1,81	5,06	3,48	3,30	3,41
	51		ΔZ	-95,337	-95,337	0	25,96	51,10	4,85	63,32	-95,335	-95,336	1,64	0,93	0,93	1,67	4,82	5,69	5,38	6,26
	52	5002 5003	ΔX	-166,984	-166,987	3	26,70	34,27	3,25	34,30	-166,986	-166,986	-1,81	-1,73	-1,73	-1,73	4,84	5,04	4,77	4,60
	53		ΔY	-23,197	-23,209	12	25,23	15,51	1,47	18,84	-23,201	-23,200	-4,13	-2,79	-2,79	-4,31	4,79	3,53	3,34	3,41
	54		ΔZ	105,378	105,382	-4	26,07	42,11	3,99	63,32	105,381	105,380	2,92	2,33	2,33	2,95	4,81	5,78	5,47	6,26
	55	5003 5004	ΔX	-174,208	-174,203	-5	26,77	23,67	2,24	34,30	-174,210	-174,209	-2,35	-1,32	-1,32	-2,19	4,66	4,34	4,11	4,39
	56		ΔY	244,440	244,437	3	27,50	11,70	1,11	18,84	244,439	244,439	-1,06	-1,27	-1,27	-0,92	4,66	3,10	2,94	3,26
	57		ΔZ	104,308	104,324	-16	26,05	30,75	2,92	63,32	104,313	104,310	4,67	1,66	1,66	4,85	4,61	5,06	4,79	5,97
	58	5004 5005	ΔX	-31,561	-31,589	28	25,32	57,59	5,46	34,30	-31,570	-31,570	-9,13	-9,28	-9,28	-9,83	4,59	5,46	5,17	4,37
	59		ΔY	60,996	61,008	-12	25,61	35,77	3,39	18,84	61,000	61,000	4,49	3,56	3,56	4,56	4,54	4,16	3,93	3,24
	60		ΔZ	14,301	14,285	16	25,14	58,53	5,55	63,32	14,297	14,296	-4,07	-5,35	-5,35	-4,39	4,56	6,13	5,80	5,93
	61	5005 5006	ΔX	125,575	125,588	-13	26,27	47,11	4,47	34,30	125,575	125,576	0,38	0,66	0,66	0,69	4,66	5,18	4,90	4,39
	62		ΔY	334,291	334,296	-5	28,46	23,03	2,18	18,84	334,294	334,293	2,63	2,17	2,17	2,58	4,75	3,82	3,52	3,26
	63		ΔZ	-191,895	-191,895	0	26,96	58,07	5,51	63,32	-191,897	-191,895	-2,06	-0,25	-0,25	-2,02	4,65	6,11	5,78	5,97
	64	5006 5007	ΔX	211,092	211,088	4	27,16	13,06	1,24	34,30	211,091	211,091	-1,21	-0,59	-0,59	-1,10	4,88	3,41	3,23	4,60
	65		ΔY	103,132	103,124	8	26,04	8,86	0,84	18,84	103,130	103,131	-1,61	-0,77	-0,77	-1,81	4,92	2,78	2,63	3,41
	66		ΔZ	-205,564	-205,567	3	27,10	25,16	2,39	63,32	-205,566	-205,564	-1,71	-0,39	-0,39	-1,67	4,86	4,65	4,40	6,26

Coordinate estimates of determined points $\hat{C}$ (Tab. 2) will be obtained using approximate values of point coordinates C° (in order $X^\circ, Y^\circ, Z^\circ$) and estimates of coordinates additions to approximate coordinates $d\hat{C}$:

$$\underset{(k,1)}{\hat{C}} = C^\circ + d\hat{C} = \begin{pmatrix} \hat{X}_1 \\ \hat{Y}_1 \\ \hat{Z}_1 \\ \vdots \\ \hat{X}_b \\ \hat{Y}_b \\ \hat{Z}_b \end{pmatrix} = \begin{pmatrix} X^\circ{}_1 \\ Y^\circ{}_1 \\ Z^\circ{}_1 \\ \vdots \\ X^\circ{}_b \\ Y^\circ{}_b \\ Z^\circ{}_b \end{pmatrix} + \begin{pmatrix} d\hat{X}_1 \\ d\hat{Y}_1 \\ d\hat{Z}_1 \\ \vdots \\ d\hat{X}_b \\ d\hat{Y}_b \\ d\hat{Z}_b \end{pmatrix} \qquad (17)$$

On the basis of coordinate estimates of determined points, a solution with cofactors obtained from the covariance matrix with $s_0^2 = RMS$ has proven to be a solution providing coordinate estimates of determined points $\hat{C}$ with the least deviations $s_{\hat{C}}$. For both eras we can determine, using standard deviations of estimates of determined coordinate points:

– the mean spatial error $s_{\hat{X}\hat{Y}\hat{Z}i}$, the average spatial error $\bar{s}_p$ (Tab. 3):

$$s_{pi} = \sqrt{s_{\hat{X}i}^2 + s_{\hat{Y}i}^2 + s_{\hat{Z}i}^2}, \qquad \bar{s}_p = \frac{1}{b}\sum_{i=1}^{b} s_{pi} \qquad (18)$$

– the mean coordinate error $s_{\hat{X}\hat{Y}\hat{Z}i}$, the average coordinate error $\bar{s}_{\hat{X}\hat{Y}\hat{Z}}$ (Tab. 3):

$$s_{\hat{X}\hat{Y}\hat{Z}i} = \sqrt{\frac{s_{\hat{X}i}^2 + s_{\hat{Y}i}^2 + s_{\hat{Z}i}^2}{3}} = \frac{s_{pi}}{\sqrt{3}}, \quad \bar{s}_{\hat{X}\hat{Y}\hat{Z}} = \frac{1}{k}\sum_{i=1}^{k} s_{\hat{X}\hat{Y}\hat{Z}i} \quad (19)$$

For improved visualisation of results, graphic evaluation was also performed using absolute confidence ellipsoids which are shown for individual processing in Tab. 4 Tab. 5.

Table 2. Coordinates of bivariant adjustment of a network in eras 04/08;
q: 1.) 5mm+1 ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE

Tabela 2. Koordynaty dopasowania biwariantowego sieci w punkatach 04/08;
q: 1.) 5mm+1 ppm, 2.) s02=1, 3.) s02=RSM, 4.) MINQUE

era	point number		C° [m]	$d\hat{C}$ [mm]				$s_{\hat{C}}$ [mm]				$\hat{C}$ [m]			
				1	2	3	4	1	2	3	4	1	2	3	4
2004	5002	X	3 941 063,356	1,69	2,66	2,66	1,72	4,79	4,35	4,16	4,60	3 941 063,358	..3,359	..3,359	..3,358
		Y	1 427 021,984	-0,51	-0,21	-0,21	-0,50	4,86	3,48	3,33	3,41	1 427 021,983	..1,984	..1,984	..1,983
		Z	4 792 984,564	2,28	3,52	3,52	2,31	4,80	4,87	4,66	6,26	4 792 984,566	..4,568	..4,568	..4,566
	5003	X	3 940 896,384	-2,54	-2,69	-2,69	-2,57	4,26	3,52	3,36	4,02	3 940 896,381	..6,381	..6,381	..6,381
		Y	1 426 998,780	3,01	3,93	3,93	3,00	4,27	2,96	2,83	2,98	1 426 998,783	..8,784	..8,784	..8,783
		Z	4 793 089,959	-5,42	-5,42	-5,42	-5,39	4,24	4,10	3,92	5,47	4 793 089,954	..9,954	..9,954	..9,954
	5004	X	3 940 722,165	4,73	3,79	3,79	4,58	4,22	3,08	2,95	3,94	3 940 722,170	..2,169	..2,169	..2,170
		Y	1 427 243,220	0,52	0,72	0,72	0,50	4,09	2,54	2,43	2,92	1 427 243,221	..3,221	..3,221	..3,220
		Z	4 793 194,256	6,62	5,98	5,98	6,53	4,17	4,31	4,12	5,35	4 793 194,263	..4,262	..4,262	..4,263
	5005	X	3 940 690,592	-2,82	-5,68	-5,68	-2,70	4,23	3,48	3,33	3,94	3 940 690,589	..0,586	..0,586	..0,589
		Y	1 427 304,207	-0,61	-1,73	-1,73	-0,50	4,13	2,79	2,67	2,92	1 427 304,206	..4,205	..4,205	..4,206
		Z	4 793 208,557	-10,29	-13,34	-13,34	-10,03	4,19	4,43	4,24	5,35	4 793 208,547	..8,544	..8,544	..8,547
	5006	X	3 940 816,181	-1,77	-4,52	-4,52	-1,68	4,30	3,57	3,41	4,02	3 940 816,179	..6,176	..6,176	..6,179
		Y	1 427 638,529	-4,09	-6,05	-6,05	-4,00	4,38	2,61	2,50	2,98	1 427 638,525	..8,523	..8,523	..8,525
		Z	4 793 016,655	12,17	13,77	13,77	12,39	4,23	4,08	3,90	5,47	4 793 016,667	..6,669	..6,669	..6,667
	5007	X	3 941 027,266	0,60	-1,33	-1,33	0,66	4,83	3,69	3,53	4,60	3 941 027,267	..7,265	..7,265	..7,267
		Y	1 427 741,651	1,57	0,70	0,70	1,50	5,06	3,03	2,90	3,41	1 427 741,653	..1,652	..1,652	..1,652
		Z	4 792 811,098	-5,79	-7,04	-7,04	-5,81	4,82	4,53	4,33	6,26	4 792 811,092	..1,091	..1,091	..1,092
2008	5002	X	3 941 063,361	-1,73	-3,27	-3,27	-1,73	4,79	5,01	5,30	4,60	3 941 063,359	..3,358	..3,358	..3,359
		Y	1 427 021,991	-4,44	-5,14	-5,14	-4,31	4,86	3,81	4,02	3,41	1 427 021,987	..1,986	..1,986	..1,987
		Z	4 792 984,570	2,89	3,93	3,93	2,95	4,80	6,05	6,40	6,26	4 792 984,573	..4,574	..4,574	..4,573
	5003	X	3 940 896,374	-0,54	-2,00	-2,00	-0,46	4,26	4,57	4,83	4,02	3 940 896,373	..6,372	..6,372	..6,374
		Y	1 426 998,782	3,43	4,07	4,07	3,39	4,27	3,45	3,65	2,98	1 426 998,785	..8,786	..8,786	..8,785
		Z	4 793 089,952	1,80	2,26	2,26	1,90	4,24	5,66	5,98	5,47	4 793 089,954	..9,954	..9,954	..9,954
	5004	X	3 940 722,171	-7,89	-8,32	-8,32	-7,65	4,22	4,43	4,68	3,94	3 940 722,163	..2,163	..2,163	..2,163
		Y	1 427 243,219	5,37	5,80	5,80	5,47	4,09	3,27	3,46	2,92	1 427 243,224	..3,225	..3,225	..3,224
		Z	4 793 194,276	-9,53	-12,08	-12,08	-9,24	4,17	5,62	5,94	5,35	4 793 194,266	..4,264	..4,264	..4,267
	5005	X	3 940 690,582	10,98	10,41	10,41	10,52	4,23	4,49	4,74	3,94	3 940 690,593	..0,592	..0,592	..0,593
		Y	1 427 304,227	-2,14	-2,64	-2,64	-1,97	4,13	3,44	3,63	2,92	1 427 304,225	..4,224	..4,224	..4,225
		Z	4 793 208,561	2,40	-1,43	-1,43	2,37	4,19	5,48	5,79	5,35	4 793 208,563	..8,560	..8,560	..8,563
	5006	X	3 940 816,170	-1,65	-1,93	-1,93	-1,79	4,30	4,20	4,44	4,02	3 940 816,168	..6,168	..6,168	..6,168
		Y	1 427 638,523	-4,51	-5,46	-5,46	-4,39	4,38	3,25	3,43	2,98	1 427 638,518	..8,518	..8,518	..8,519
		Z	4 793 016,666	0,35	-1,69	-1,69	0,35	4,23	5,32	5,62	5,47	4 793 016,666	..6,664	..6,664	..6,666
	5007	X	3 941 027,258	1,15	1,48	1,48	1,10	4,83	4,16	4,40	4,60	3 941 027,259	..7,259	..7,259	..7,259
		Y	1 427 741,647	1,88	1,76	1,76	1,81	5,06	3,30	3,48	3,41	1 427 741,649	..1,649	..1,649	..1,649
		Z	4 792 811,099	1,64	0,93	0,93	1,67	4,82	5,38	5,69	6,26	4 792 811,101	..1,100	..1,100	..1,101

Table 3. Mean and average coordinate and spatial errors in eras 04/08;
q: 1.) 5mm+1·D ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE

Tabela 3. Wartości średnie i średnie koordynaty oraz przestrzenne epok 04/08
q: 1.) 5mm+1·D ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE

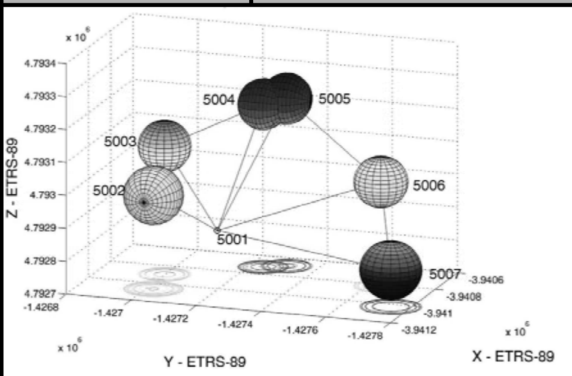
era	point number	Mean coordinate error [mm]								Mean spatial error [mm]							
		Separate processing				Bivariant processing				Separate processing				Bivariant processing			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2004	5002	4,818	4,387	4,387	4,891	4,086	4,086	4,273	4,898	8,345	7,599	7,599	8,471	7,077	7,077	7,401	8,484
	5003	4,257	3,654	3,654	4,275	3,403	3,403	3,559	4,282	7,373	6,328	6,328	7,404	5,894	5,894	6,164	7,416
	5004	4,166	3,482	3,482	4,180	3,243	3,243	3,392	4,186	7,216	6,032	6,032	7,239	5,618	5,618	5,875	7,250
	5005	4,186	3,726	3,726	4,180	3,471	3,471	3,629	4,186	7,251	6,454	6,454	7,239	6,011	6,011	6,286	7,250
	5006	4,307	3,567	3,567	4,275	3,322	3,322	3,474	4,282	7,459	6,178	6,178	7,404	5,754	5,754	6,017	7,416
	5007	4,908	3,902	3,902	4,891	3,634	3,634	3,801	4,898	8,502	6,759	6,759	8,471	6,295	6,295	6,583	8,484
2008	5002	4,813	4,905	4,905	4,906	5,329	5,329	5,043	4,898	8,336	8,495	8,495	8,497	9,229	9,229	8,735	8,484
	5003	4,252	4,522	4,522	4,288	4,913	4,913	4,650	4,282	7,366	7,832	7,832	7,427	8,510	8,510	8,053	7,416
	5004	4,162	4,418	4,418	4,192	4,800	4,800	4,543	4,186	7,209	7,653	7,653	7,262	8,314	8,314	7,869	7,250
	5005	4,182	4,420	4,420	4,192	4,803	4,803	4,545	4,186	7,244	7,656	7,656	7,262	8,318	8,318	7,872	7,250
	5006	4,302	4,221	4,221	4,288	4,586	4,586	4,340	4,282	7,452	7,310	7,310	7,427	7,942	7,942	7,517	7,416
	5007	4,904	4,245	4,245	4,906	4,612	4,612	4,365	4,898	8,493	7,353	7,353	8,497	7,989	7,989	7,560	8,484
era		Average coordinate error								Average spatial error							
2004	sieť	4,440	3,786	3,786	4,449					7,691	6,558	6,558	7,705				
2008	sieť	4,436	4,455	4,455	4,462					7,683	7,717	7,717	7,729				
2004+2008	sieť					4,184	4,184	4,135	4,455					7,246	7,246	7,161	7,717

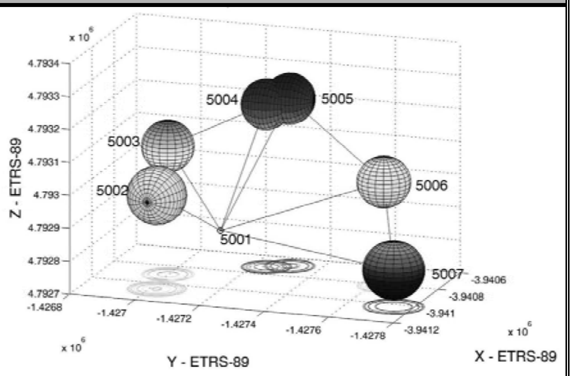
Table 4. Separate network adjustment, Selection of cofactors

Tabela 4. Dopasowanie sieci rozdzielnych. Wybór co-faktorów

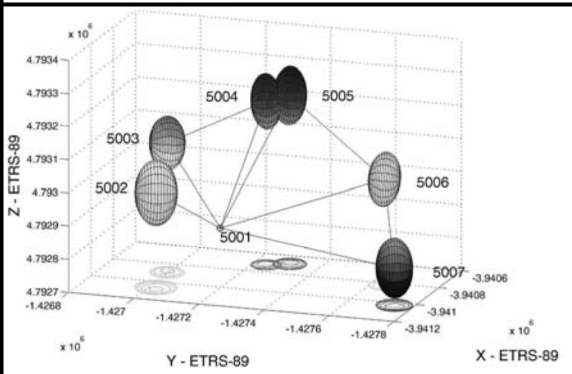
Epoch 2004 | **Epoch 2008**

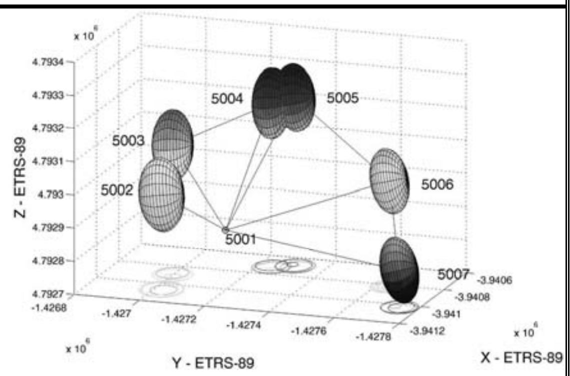
in accordance with 5 mm + 1·D ppm



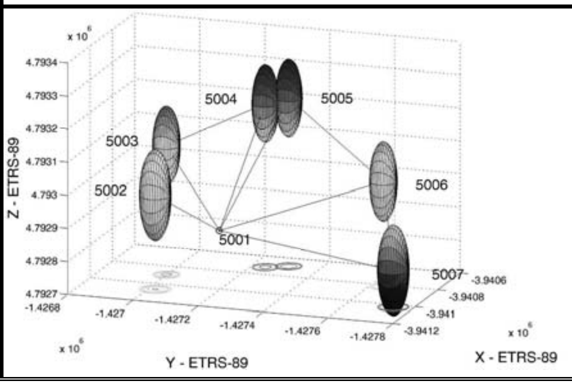


cov. matrix from SS, s_0^2=RMS, s_0^2=1





estimated using the MINQUE method



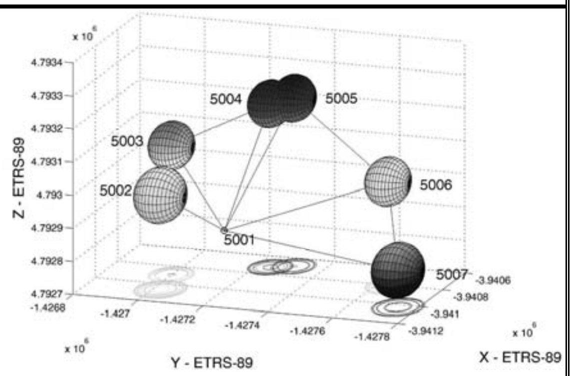
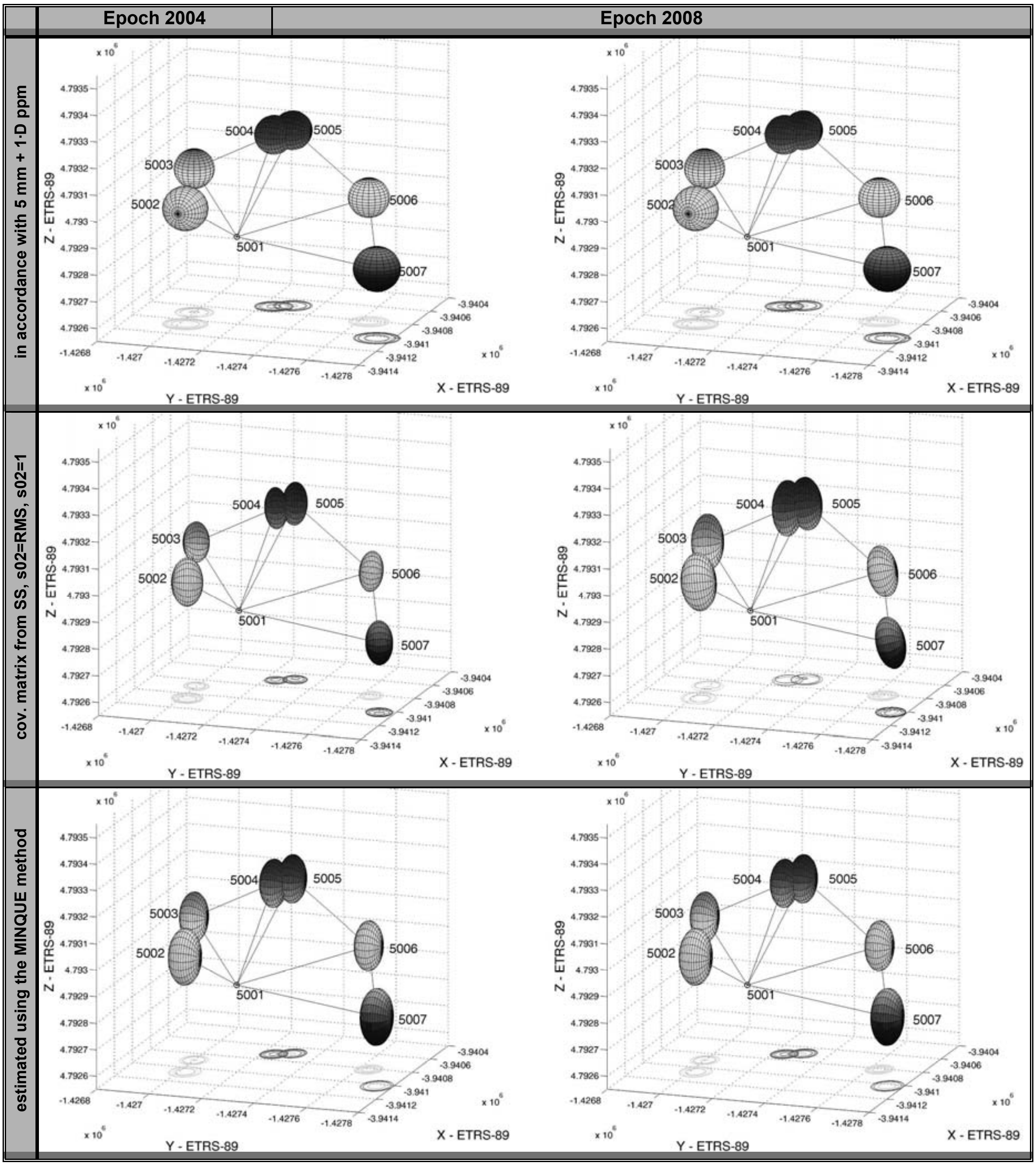


Table 5. Bivariant network adjustment, Selection of cofactors

Tabela 5. Dopasowanie sieci biwariantowej. Wybór co-faktorów



Analysis of deformation vector

Before analysis of the deformation vector, the homogeneity of networks of eras 2004 and 2008 was verified during separate processing using Fisher's test of congruence on two files with differing accuracy. Since the measurements were carried out with approximately the same level of accuracy, it is possible to merge them into one model. The MINQUE method mainly proved to be advantageous during separate processing, using which value of the nominal a posteriori factor $s_0^2 = 1$ in both eras was ascertained. Test values are stated in Tab. 6.

Table 6. Testing congruency of two files with differing accuracy

Tabela 6. Testowanie zbieżność dwóch plików o różnej dokładności

Fisher's congruency test		Testing statistics T	Critical value T_{krit}
Cofactor	5 mm + 1·D ppm	1.002	2.403
	Cov. matrix from SS, s_0^2 = RMS	1.114	
	Cov. matrix from SS, $s_0^2 = 1$	1.361	
	Estimated using MINQUE method	1.000	

During deformation survey, differences in coordinates $\Delta\hat{C}_i$ of individual points determined in 2004 and 2008 were measured within separate and bivariant adjustment using all types of cofactors in general form:

$$^{04/08}\Delta\hat{C}_i = {}^{2008}\hat{C}_i - {}^{2004}\hat{C}_i. \quad (1)$$

Differences in the direction of individual axes were numerically expressed:

$$\Delta\hat{X} = {}^{t_{i+1}}\hat{X} - {}^{t_i}\hat{X},\ \Delta\hat{Y} = {}^{t_{i+1}}\hat{Y} - {}^{t_i}\hat{Y},\ \Delta\hat{Z} = {}^{t_{i+1}}\hat{Z} - {}^{t_i}\hat{Z} \quad (2)$$

as well as differences:

$$\Delta\hat{X}\hat{Y} = \sqrt{\Delta\hat{X}^2 + \Delta\hat{Y}^2},\ \Delta\hat{Y}\hat{Z} = \sqrt{\Delta\hat{Y}^2 + \Delta\hat{Z}^2},$$
$$\Delta\hat{X}\hat{Z} = \sqrt{\Delta\hat{X}^2 + \Delta\hat{Z}^2} \quad (22)$$

in the appropriate planes and spatial differences:

$$\Delta\hat{X}\hat{Y}\hat{Z} = \sqrt{\Delta\hat{X}^2 + \Delta\hat{Y}^2 + \Delta\hat{Z}^2}. \quad (3)$$

Since from a numeric viewpoint no significant difference was discovered in the differences between separate and bivariant processing, individual differences stated in Tab. 7 to Tab. 9 are stated from separate adjustment. Deformation vector $\Delta\hat{C}_i$ was first verified by a global congruency test and then also using a localization congruency test (Weiss, 2007).

Difference values between individual processing methods from the viewpoint of cofactor selection are varied. However, testing using Fisher's test showed, in all cases, a shift of the same point – 5005 – in axis direction Y and Z. In Y axis coordinate (Tab. 7) the greatest shift is shown when using covariance matrix with a nominal a priori variation factor $s_0^2 = RMS$, as well as $s_0^2 = 1$, where $\Delta\hat{Y} = 19{,}04\,mm$. The least shift was shown by using $5\,mm + 1 \cdot D\,ppm$, $\Delta\hat{Y} = 18{,}47\,mm$. The MINQUE method estimated the shift at $\Delta\hat{Y} = 18{,}52\,mm$. On the other hand, in Z, axis coordinate the greatest shift is shown when using $5\,mm + 1 \cdot D\,ppm$, $\Delta\hat{Z} = 16{,}69\,mm$. The least shift was shown by using a covariance matrix with $s_0^2 = RMS$, as well as $s_0^2 = 1$, $\Delta\hat{Z} = 15{,}91\,mm$. The MINQUE method shows the coordinate difference of point 5005 situated between values from previous methods.

When surveying whether this shift took place or the differences are simply an accumulation of sighting errors in planes XY and YZ of point 5005, compliance of all used methods was again confirmed. Fisher's test was used for the evaluation. The difference between numeric values is around 1 millimetre. The differences were shown when surveying the shift in plane XZ (Tab. 8), where three methods indicated differences as an accumulation of sighting

Table 7. Differences 1D in the direction of individual axes X, Y, Z

Tabela 7. Różnice 1D w kierunku pojedynczych osi X, Y, Z

Point number	Differences of coordinates	Selection of cofactors							
		5 mm + 1·D ppm		Cov. matrix from SS, s_0^2 = RMS		Cov. matrix from SS, s_0^2 = 1		estimated MINQUE method	
		shift	T	shift	T	shift	T	shift	T
5002	$\Delta\hat{X}$	1.588	0.055	-0.932	0.020	-0.932	0.019	1.556	0.057
	$\Delta\hat{Y}$	3.068	0.199	2.066	0.160	2.066	0.156	3.194	0.438
	$\Delta\hat{Z}$	6.610	0.950	6.407	0.680	6.407	0.655	6.646	0.564
5003	$\Delta\hat{X}$	-8.001	1.761	-9.312	2.606	-9.312	2.503	-7.889	1.921
	$\Delta\hat{Y}$	2.414	0.160	2.135	0.220	2.135	0.214	2.389	0.321
	$\Delta\hat{Z}$	0.227	0.001	0.676	0.009	0.676	0.009	0.292	0.001
5004	$\Delta\hat{X}$	-6.625	1.230	-6.109	1.281	-6.109	1.220	-6.222	1.250
	$\Delta\hat{Y}$	3.847	0.442	4.082	0.971	4.082	0.933	3.972	0.927
	$\Delta\hat{Z}$	3.852	0.426	1.934	0.075	1.934	0.072	4.229	0.313
5005	$\Delta\hat{X}$	3.798	0.403	6.089	1.149	6.089	1.105	3.222	0.335
	$\Delta\hat{Y}$	18.471	9.980	19.094	18.629	19.094	17.978	18.528	20.178
	$\Delta\hat{Z}$	16.691	7.954	15.910	5.097	15.910	4.917	16.396	4.703
5006	$\Delta\hat{X}$	-10.877	3.205	-8.411	2.327	-8.411	2.255	-11.111	3.811
	$\Delta\hat{Y}$	-6.417	1.071	-5.410	1.687	-5.410	1.627	-6.389	2.293
	$\Delta\hat{Z}$	-0.820	0.019	-4.456	0.442	-4.456	0.424	-1.042	0.018
5007	$\Delta\hat{X}$	-7.453	1.192	-5.196	0.872	-5.196	0.849	-7.555	1.346
	$\Delta\hat{Y}$	-3.686	0.265	-2.932	0.429	-2.932	0.419	-3.694	0.586
	$\Delta\hat{Z}$	8.427	1.526	8.966	1.624	8.966	1.573	8.479	0.919
Critical value:		**Tcrit = 4.543**							

errors, only when substituting cofactors in accordance with $5\,mm + 1 \cdot D\ ppm$ was this difference in the plane marked as a deformation. If the deformation in the 3D space is verified, all methods of substituting cofactors confirmed that the deformation existed on point 5005 (Tab. 9), while the numerical difference is $\pm$ 0,7 mm.

For better visualisation, sighting errors for the deformation network are surveyed using graphic methods via absolute confidence ellipsoids obtained from adjustment in eras 2004 and 2008 and differences of point in the area of era 2008 against era 2004 via relative confidence ellipsoids. For comparison, two points of the deformation network were selected which could be compared – this was point 5004, where no shift took place and point 5005 on which a shift took place. Tab. 10 shows point 5004 of the example network sighted in era 2004 and 2008 with drawn absolute and relative confidence ellipsoids which represent 95% of the area of occurrence of the given point in the appropriate era. Tab. 11 shows point 5005 with drawn absolute and relative confidence ellipsoids serving for graphic survey of deformities.

As is clear from Tab. 8 the absolute confidence ellipsoids of point 5004 have mutual intersections and the juncture of their centres does not exceed the surface of the relative confidence ellipsoid; there-

Table 8. Differences 2D in axes XY, YZ, XZ

Tabela 8. Różnice 2D w kierunku osi XY, YZ, XZ

Point number	Differences of coordinates	Selection of cofactors							
		5 mm + 1·D ppm		Cov. matrix from SS, s_0^2 = RMS		Cov. matrix from SS, s_0^2 = 1		estimated MINQUE method	
		shift	T	shift	T	shift	T	shift	T
5002	$\Delta\hat{X}\hat{Y}$	3.455	0.127	2.267	0.101	2.267	0.098	3.553	0.248
5003		8.357	0.961	9.553	1.572	9.553	1.506	8.243	1.121
5004		7.661	0.836	7.347	1.309	7.347	1.248	7.382	1.089
5005		18.857	5.191	20.042	9.396	20.042	9.071	18.806	10.257
5006		12.629	2.138	10.001	1.805	10.001	1.753	12.817	3.052
5007		8.315	0.729	5.966	0.573	5.966	0.560	8.410	0.966
5002	$\Delta\hat{Y}\hat{Z}$	7.287	0.575	6.732	0.385	6.732	0.371	7.374	0.501
5003		2.425	0.081	2.240	0.111	2.240	0.107	2.407	0.161
5004		5.444	0.434	4.517	0.492	4.517	0.473	5.802	0.620
5005		24.895	8.967	24.854	10.590	24.854	10.178	24.741	12.441
5006		6.469	0.545	7.009	0.937	7.009	0.897	6.473	1.156
5007		9.198	0.895	9.433	1.284	9.433	1.259	9.249	0.752
5002	$\Delta\hat{X}\hat{Z}$	6.798	0.502	6.474	0.376	6.474	0.361	6.825	0.311
5003		8.004	0.881	9.336	1.478	9.336	1.423	7.894	0.961
5004		7.663	0.828	6.407	0.861	6.408	0.827	7.523	0.782
5005		17.118	4.178	17.036	2.716	17.036	2.626	16.710	2.519
5006		10.908	1.612	9.518	1.258	9.518	1.223	11.160	1.915
5007		11.250	1.359	10.363	1.471	10.363	1.413	11.357	1.133
Critical value:		**Tcrit = 3.682**							

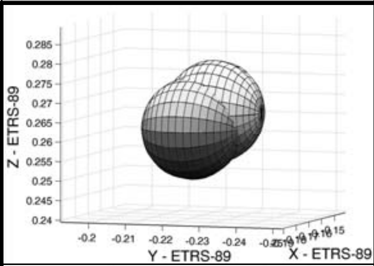
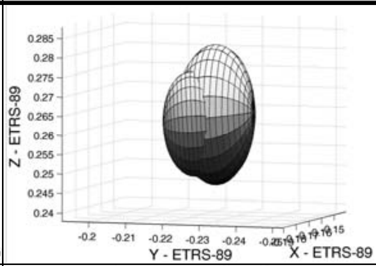
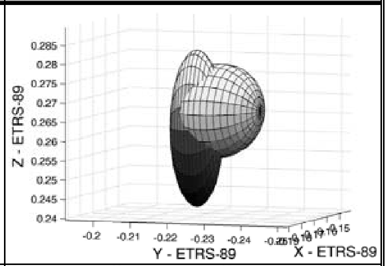
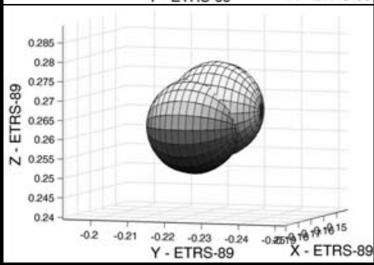
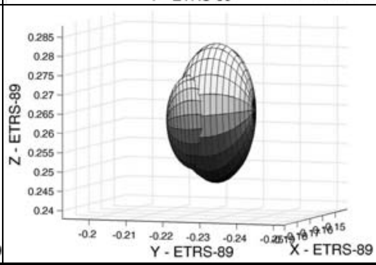
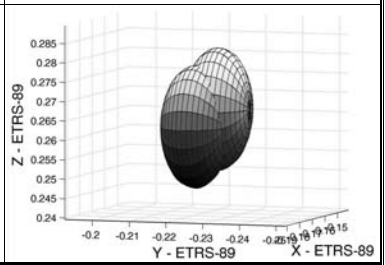
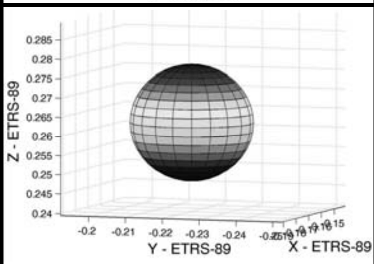
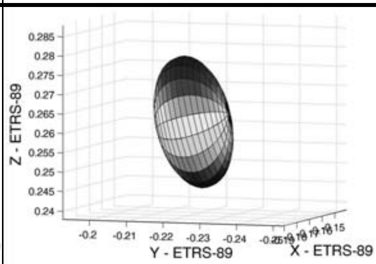
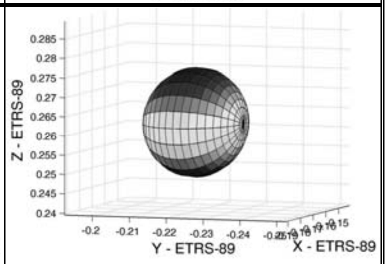
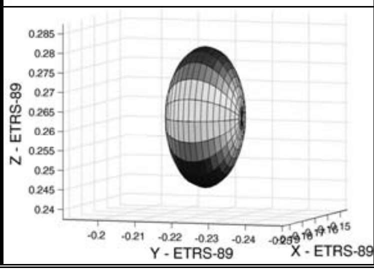
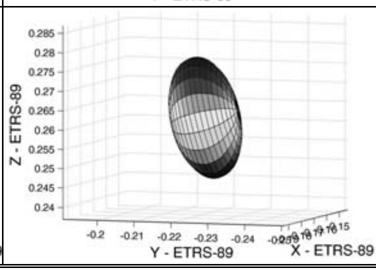
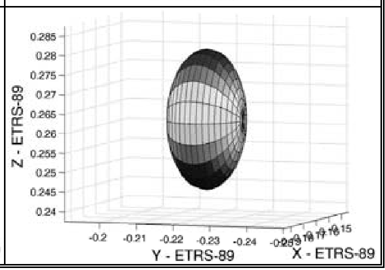
Table 9. Differences 3D in area XYZ

Tabela 9. Różnice 3D na powierzchni XYZ

Point number	Differences of coordinates	Selection of cofactors							
		5 mm + 1·D ppm		Cov. matrix from SS, s_0^2 = RMS		Cov. matrix from SS, s_0^2 = 1		estimated MINQUE method	
		shift	T	shift	T	shift	T	shift	T
5002	$\Delta\hat{X}\hat{Y}\hat{Z}$	7.458	0.401	6.796	0.289	6.796	0.278	7.536	0.353
5003		8.360	0.641	9.577	1.141	9.577	1.094	8.248	0.748
5004		8.575	0.699	7.597	0.964	7.597	0.922	8.508	0.830
5005		25.183	6.112	25.589	7.062	25.589	6.789	24.950	8.406
5006		12.655	1.432	10.948	1.224	10.948	1.187	12.859	2.041
5007		11.838	0.994	10.770	1.218	10.770	1.184	11.943	0.950
Critical value:		**Tcrit = 3.287**							

Table 10. Graphic visualisation of point 5004 via confidence ellipsoids when selecting various types of cofactors

Tabela 10. Graficzna wizualizacja punktu 5004 poprzez elipsoidy ufności przy wyborze różnych typów co-faktorów

Eras 2004/2008		Selection of cofactors with graphic visualisation for point 5004		
		5 mm + 1·D ppm	Cov. matrix from SS, s_0^2=RMS , s_0^2=1	Estimated using MINQUE method
Absolute confidence ellipsoids	Separate adjustment			
	Bivariant adjustment			
Relative confidence ellipsoids	Separate adjustment			
	Bivariant adjustment			

fore, it is possible to state that a shift of the surveyed point did not take place. Tab. 11 shows that the absolute confidence ellipsoids of point 5005 do not have mutual intersections and juncture of their centres exceeds the surface of the relative confidence ellipsoid; therefore, it is possible to state that a shift of the surveyed point took place.

Summary

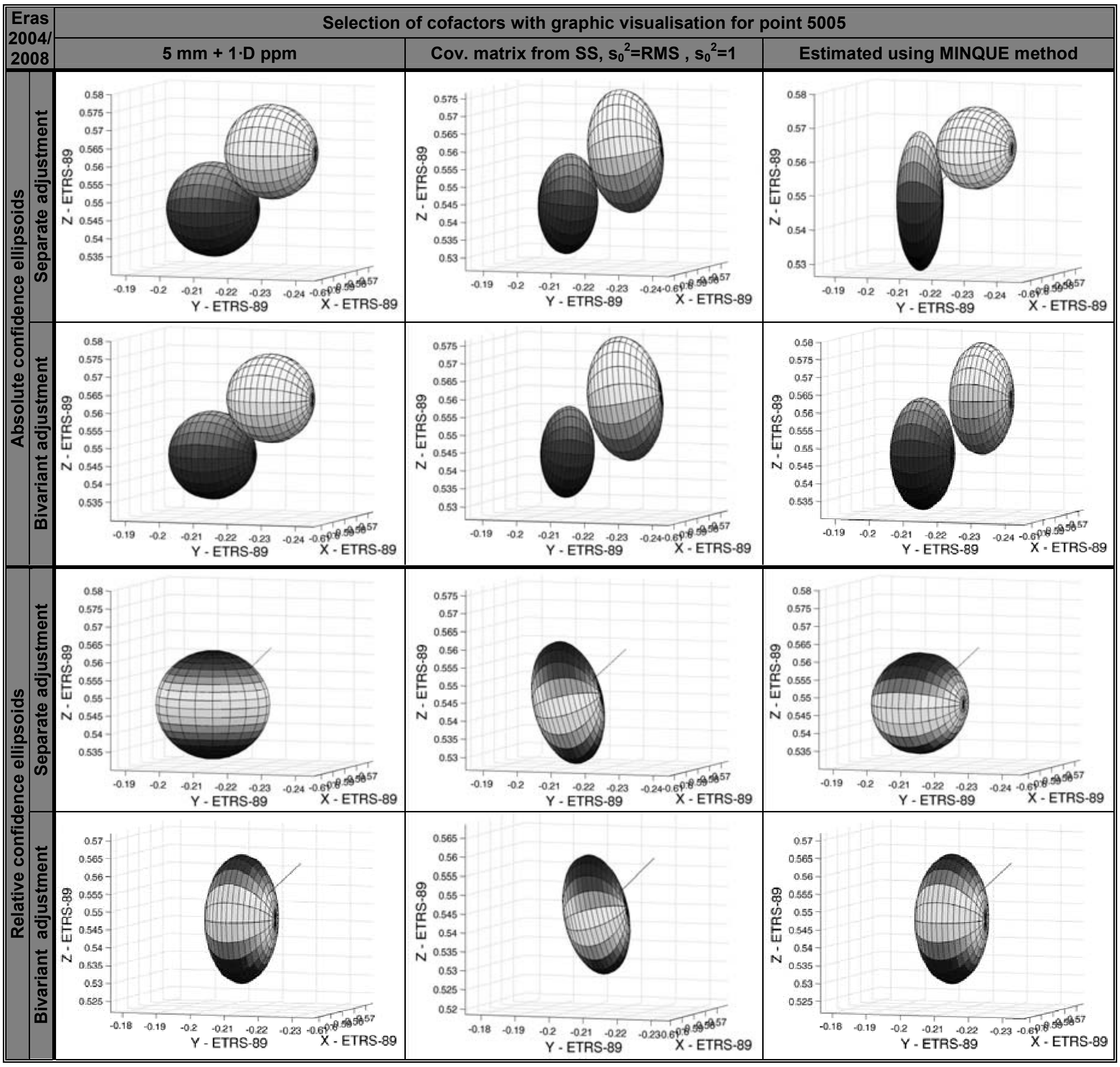
Determining the characteristics of point accuracies, sighted using GNSS technology on the basis of a posteriori empiric characteristics, is advantageous because all primary data from observations is always available.

The submitted work addresses deformation analysis of a network sighted using GNSS at WTPS Cierny Vah in two eras: 2004 and 2008. The observed values were primarily processed in the SS software environment. Via the Trimble Geomatic Office software environment, coordinates of the observed points were transferred to the ETRS-89 coordinate system which takes into account the movement of the Eurasian lithospheric plate and hence the shift against the WGS-84 system.

The Gauss-Markov model with full rank was used for processing of adjustment of parameter measurement. Observations were processed and adjusted using four types of varying cofactors (weights) in accordance with the recommendations of the GNSS receiver producer: $5\,mm + 1 \cdot D\,ppm$, via substitution of a covariance matrix from the SS, where as an a priori nominal variance factor, the following data was used $s_0^2 = RMS$ and constant $s_0{}^2 = 1$. In the fourth case, cofactors (weights) of observation components were also estimated using the MINQUE method. Eras 2004 and 2008 were adjusted separately as well as bivariantly. From the

Table 11. Graphic visualisation of point 5005 via confidence ellipsoids when selecting various types of cofactors

Tabela 11. Graficzna wizualizacja punktu 5005 poprzez elipsoidy ufności przy wyborze różnych typów co-faktorów



viewpoint of the number of processed eras, significant differences between processing results were not found; therefore, it is up to the person resolving the issue as to which of these methods is selected, under the condition that both eras are processed with approximately the same level of accuracy.

From the viewpoint of standard deviations, the solution using covariance matrixes from the SS proved to be most accurate. When using the MINQUE method, weights were estimated on the basis of the accuracy recommended by the producer, while numeric results from processing showed that the use of the MINQUE method is a suitable alternative to the demanding inputting of covariance matrixes in the SS software environment. Via application of the MINQUE method, the greatest weights were estimated and assigned to vectors in the direction of the *Y* axis, then *X* and the least in the direction of the *Z* axis. On the basis of coordinate estimates of determined points, a solution with cofactors obtained from the covariance matrix with $s_0^2 = RMS$ has proven to be a solution providing coordinate estimates of determined points $\hat{C}$ with the least deviations $s_{\hat{C}}$.

Testing using Fisher's test in four types of processing showed that shifts of point 5005 took place in the direction of axis Y and Z , in planes XY and XYZ. as well as in 3D area. YZ Graphic visualisation for two selected points of the deformation network displays absolute confidence ellipsoids obtained from adjustment in eras 2004 and 2008. A change of point positions in era 2008 against 2004 is shown via relative confidence ellipsoids. For comparison, two points of the deformation network were selected which could be compared – this was point 5004, where no shift took place and point 5005 on which a shift took place.

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Analiza zniekształceń struktury sieci mierzona za pomocą nawigacji satelitarnej z wyborem kofaktorów

Niniejsza praca dotyczy analizy deformacji geodezyjnych sieci 3D w elektrowni wodnej Cierny Vah zaobserwowanych za pomocą technologii nawigacji saterlitarnej w latach 2004 i 2008. Celem pracy jest ocena wpływu parametrów wprowadzanych do modelu, oszacowanie parametrów pierwszego i drugiego stopnia struktury sieci oraz przedstawienie wyników analizy zniekształceń za pomocą graficznej wizualizacji poszczególnych procesów i analiz. Cztery typy kofaktorów zostały użyte do przetworzenia i dostosowania obserwacji zgodnie ze wskazówkami producenta odbiornika GNSS (Global Navigation Satellite Systems) poprzez zastąpienie macierzy kowariancji z pomiaru widma przy użyciu RMS oraz stałej 1 jako wariancji nominalnej. Metoda MINQUE została również zastosowana w celu określenia kofaktorów komponentów obserwacyjnych. Największe wagi zostały oszacowane przez zastosowanie metody MINQUE i przypisanie im wektorów. Bazując na szacunkowych współrzędnych określonych punktów, rozwiązaniem okazał się być wybór kofaktorów z użyciem macierzy kowariancji ze względu na najmniejsze odchylenia. Z punktu widzenia odchyleń standardowych, wybór macierzy kowariancji z badania widm osiągnął najwyższy stopień dokładności. Wyniki liczbowe otrzymane z przetworzenia danych pokazały, że użycie metody MINQUE jest odpowiednią alternatywą dla żmudnego wprowadzania macierzy kowariancji do środowiska programistycznego.

Słowa kluczowe: lokalna geodezyjna sieć 3D, miernictwo GNSS, LMS, MINQUE, analiza zniekształceń, elipsoida błędów