Attempt of Determining Optimal Values of Mineral Raw Materials Beneficiation Factors

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DOI: 10.29227/IM-2015-02-46

Abstract

The methods of determining so-called optimal conditions of mineral raw materials beneficiation were presented in the paper. The copper ore from deposits of KGHM Polska Miedź, region Polkowice as well one of hard coals originated from Upper Silesia, coal mine Janina were selected to the research. It was stated that on the basis of balance equation is possible to determine optimal characteristics of beneficiated material. To determine the optimal point, Authors used commonly known upgrading curves, i.e. Halbich, Fuerstenau and Madej curves. For all of them the criteria of selecting optimal point were accepted as the point of the largest curvature. However, it occurred that for relation between sums of recovery in concentrate and sums of residuals recovery in tailings is possible to determine the extreme point which can be treated as the technological optimum. Accepting that α = const we face the problem of determining a conditional extreme of function of sums of useful component recovery in concentrate and sums of residuals recovery in tailings of two variables β and ϑ. This methodology was used for chosen copper ore and chosen hard coal. However it occurred that selection of the optimal point in the method of sums of recoveries is also ambiguous. It is worthy to notice that selection of technological optimum for researched raw material is also evaluation of its susceptibility to beneficiation. The whole paper was ended with conclusions.

Keywords: minerals beneficiation factor, Halbich curve, Fuerstenau curve, Madej curve

Introduction

Mineral processing is based on entry material (feed) division into products – mostly two of them being called usually as concentrate and tailings. The general law of this process is the Fredholm's equation (Tumidajski, 1997; Tumidajski and Saramak, 2002):

$$\int_D p(w,x)f(w)dw = g(x) \tag{1}$$

where w is the variable of distribution described by density function $g(w)$; $p(w, x)$ is function of transfer of particles having feature w to product being characterized with value of feature x of distribution described by density function $g(x)$. By classical approach and material separation into two products we face usual balance equation in which averaged values occur: α – mean contents of the feature in the feed; β – mean contents of the feature in the concentrate and ϑ – mean contents of the feature in tailings.

In practical applications these three values allow to create further evaluations of separation process course, in particular to determine yields and recoveries of particles representing respective mean contents. Because separation of grained materials is characterized with its randomness and is not precise it can create problems connected with description of this separation or determination of separation limits. This is connected also with time of process occurring, so with value of yield γ.

Two groups of factors (variables) decide about the course of separation (beneficiation) process: structure of grained material (feed) and its preparation to the process as well conditions of the process course. First group of conditions is the basis to determine potential possibilities of recovering chosen component from the material and evaluate level of this recovery in real industrial process conditions (yields, recoveries and contents of selected component in concentrate and tailings). In many cases to analyze susceptibility of raw material to certain type of beneficiation (especially gravitational one) laboratory experiments are conducted and on their basis the beneficiation curves are created which operate only with basic values like α, β, ϑ and their calculated combinations. If analogical curves are created for real values of α, β, ϑ obtained in laboratory or industrial conditions we face so-called real beneficiation curves which can significantly differ from beneficiation curves. The important problem is then to determine so-called technological optimum which can be understood as point in the space (α, β, ϑ) which would be the natural answer of mineral raw material structure to conditions of beneficiation process course (the one where special actions with

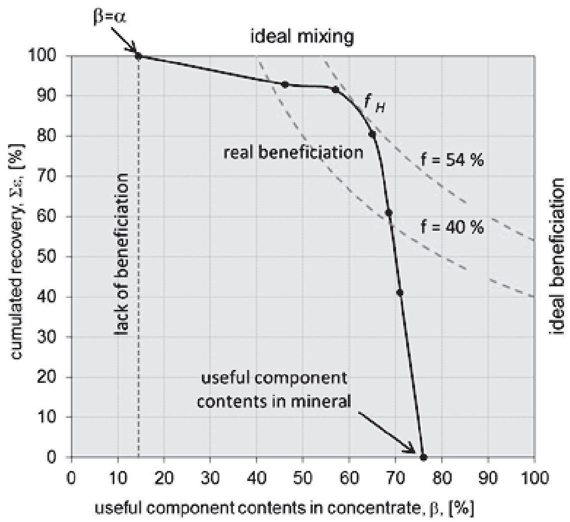


Fig. 1. Properties of Halbich's partition curve in system ε = f(*β*), general data and location of point fH, course of function f = ε*β*/100% dependably on the value of *f*

Rys. 1. Właściwości krzywej wzbogacania Halbich'a w układzie ε = f(*β*), dane ogólne i lokalizacja punkyu fH, przebieg funkcji f = ε*β*/100% w zaleznności od wartości *f*

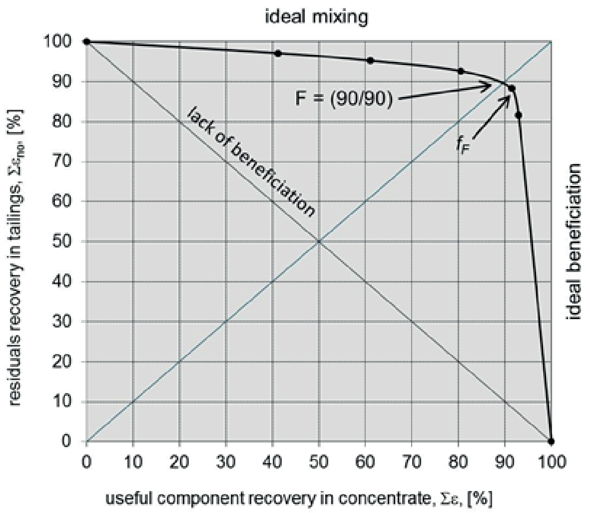


Fig. 2. Properties of Fuerstenau's curve

Rys. 2. Właściwości krzywej Fuerstenaua

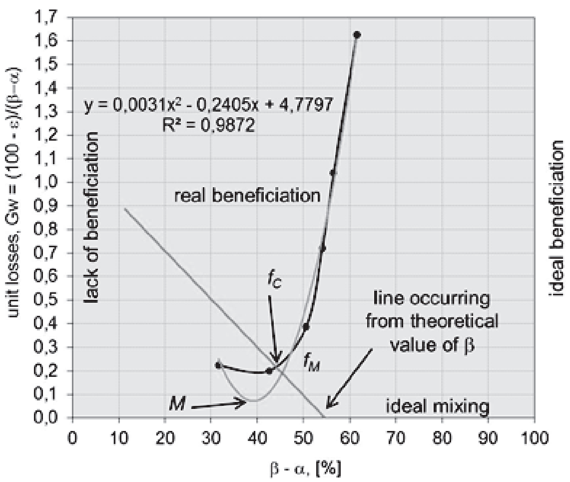


Fig. 3. Madej's partition curve: properties of the curve, location of characteristic points f_C and f_M and *M*

Rys. 3. Krzywa Madeja: właściwości krzywej, lokalizacja punktów charakterystycznych fC , fM , M purpose of changing products yields or recoveries do not occur). If we apply additional actions changing yields of recoveries with maintaining value of the component in the required product we head to obtain the economical optimum.

By creating the factors and indicating points in which the technological optimum is realized the following rule should be accepted: by certain value of α, values of β and ϑ should be respectively equal or significantly close to each other, no matter what is the type of applied beneficiation or partition curve.

The purpose of the presented considerations is the attempt of defining or more precise determining potential possibilities of the material by certain type of beneficiation and certain level of preparation of the feed to this process. These considerations will be general, which take into account only basic values characterizing the feed and its separation, α, β and ϑ as well yield γ.

Applied methods of determining optimal values of mineral raw materials beneficiation factors

So far, by presentation of the results of beneficiation conducted especially in laboratory conditions three types of benficiation curves are used usually: Henry's curves, Halbich's curves and Fuerstenau's curves (Drzymala, 2007; Foszcz, 2013; Foszcz et al., 2009; 2010; 2016). Henry's curves are usually applied (because of technical reasons – possibility of separation into density fractions) for coal (Surowiak, 2013a; b; Topiarzová and Čablík, 2011). Halbich's curve which presents the relation ε=f(β) and Fuerstanau's curve presenting relation ε_r=f(ε), where

$$\varepsilon = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}\cdot 100$$

and (2)

$$\varepsilon_r = (100-\frac{\alpha-\vartheta}{\beta-\vartheta}\cdot 100)\frac{100-\vartheta}{100\alpha}$$

are usually used by evaluation of ores flotation. Halbich's and Fuerstenau's curves do not have extreme points and that is why determination of extreme values of beneficiation factors is impossible by means of them. It is possible for these curves to try to determine starting points of dependent variable decline, like ε or ε_r or points of the biggest curvature. In first case it will be point in which the change of sign of first derivative occurs and in second one – minimization of curvature radius,

$$= \frac{(1+\left(\frac{dy}{dx}\right)^2)}{\frac{d^2y}{dx^2}}$$

where $y = \varepsilon$, $x = \beta$ for Halbich's curie or $y = \varepsilon_r$, $x = \varepsilon$ for Fuerstanau's curve. The values of first and second derivatives must be determined experimentally (on the basis of established increments of applied variables). The basis of such approach can be the interpretation of Figure 1. As it can be noticed easily the increments of value β stops dominate over declines of ϑ.

It is worthy to notice that proper evaluation of material beneficiation should take into consideration the value of ϑ, which indicates the biggest sensitivity (Foszcz et al., 2009). Among methods of evaluating technological optimum also Madej's curve should be considered which is the representation of the relation

$$G_w = \frac{100-\varepsilon}{\beta-\alpha} \qquad (3)$$

and has minimum which can be interpreted as technological optimum.

Let's discuss the applications of Halbich's, Fuerstenau's and Madej's curves, respectively.

Halbich's curve

From properties of Halbich's curve it occurs that the characteristic point of separation is point of the biggest convexity (biggest curvature) which is the point f_H (Halbich, 1934). According to the formula determining curvature radius it is point in which second derivative of recovery calculated according to the contents in concentrate achieves minimum. According to Kelly and Spottiswood (Kelly and Spottiswood, 1982) this point can be determined on the basis of maximum value of parameter f = $\varepsilon\beta$/100 for which we achieve tangential curve to Halbich's curve; in example shown on Fig. 1 is occurs for the value f = 54%. In this point the dynamics of not useful component flow to concentrate starts to dominate over dynamics of useful component flow to this concentrate. The approximated coordinates of the optimal point can be achieved by means of experimental method by very detailed conductance of the flotation experiments.

Fuerstenau's curve

The properties of Fuerstenau's curves for hypothetical data presented on Fig. 1 were presented on Fig. 2. The optimal point, similarly as on Halbich's curve, can be point of the biggest convexity fF. It is the point in which more gangue occur in concentrate than useful components

The Fuerstenau's curve allows also to determine technologically optimal quality of the concentrate in other way. Either the point of the biggest convexity f_F is determined as in case of Halbich's curves, either the optimal point F is determined (Drzymala and Ahmed, 2005) which lies on intersection of real beneficiation curve and diagonal of identity of useful component recovery in concentrate and residuals recovery in tailings. In optimal point F the recovery of considered component in concentrate is equal to residuals recovery of remaining components in tailings.

It is needed to add that generally Fuerstenau's curve is some kind of analog to Halbich's curve but is scaled in the way that removes influences of feed quality variation on beneficiation. Thanks to it, Fuerstenau's curve has constant location of the lines of lack of beneficiation and ideal beneficiation, no matter what is the useful component contents in the feed. Because of that, the axis of β is replaced with axis of residuals recovery in tailings ε_r.

Madej's curve

The properties of Madej's curve in system (β-α, unit losses), where unit losses

$$G_w = \frac{\eta}{(\beta-\alpha)} = \frac{100-\varepsilon}{\beta-\alpha}$$

were shown on Fig. 3. It is quite complicated partition curve. Fig. 3 shows (also for hypothetical data presented on Figs. 1 and 2) ways of appropriate deteremination of separation optimal point f_M (biggest curvature) as well so-called central point f_C on example of beneficiation products. It supposed to be notices that according to Madej (Madej, 1978), on the bassis of partition curve

$$\frac{\eta}{(\beta-\alpha)} = f(\beta-\alpha)$$

is possible to determine also other optimal point which fulfills other criteria. This is the point obtained as a result of approximation of real beneficiation points by parabolic function and determination of its minimal point which was marked on the Fig. 3 as M.

Separation efficiency factor based on sum of recoveries of useful component in concentrate as residual substances in tailings

I occurred that the sum of recovery of useful component and residuals recovery ε_r has extreme which can be treated as technological optimum.

Let treat this case generally.

So, we have:

$$s = \varepsilon + \varepsilon_r = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}\cdot 100 + \left(100 - 100\cdot\frac{\alpha-\vartheta}{\beta-\vartheta}\frac{100-\vartheta}{\beta-\vartheta}\right)\frac{100-\vartheta}{100-\alpha} =$$

$$100\left[\frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha} - \frac{100-\vartheta}{100-\alpha} - \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{100-\vartheta}{100-\alpha}\right] \quad (4)$$

It is worthy to notice that equation (4) can be led to the form

$$\varepsilon + \varepsilon_r = -100\frac{100-\vartheta}{100-\alpha} + \varepsilon - \varepsilon^2\frac{\alpha^2(100-\alpha)(\beta-\vartheta)}{100\beta^2(100-\alpha)(\alpha-\vartheta)} \quad (5)$$

From the geometrical point of view this equation represent hypersurface in 4-dimensional space with formal limitations $0\le\alpha\le$; $0\le\beta\le 100$; $0\le\vartheta\le 100$ and $\beta>\alpha$ i $\vartheta<\alpha$. In equation (4) the limitation being also condition was already used, which was so-called balance equation.

$$\alpha = \frac{\alpha-\vartheta}{\beta-\vartheta}\beta + \left(1-\frac{\alpha-\vartheta}{\beta-\vartheta}\right)\vartheta \quad (6)$$

If we accept that α=const then we face the problem of determining conditional extreme of two variables (β and ϑ) function s.

The plotting of this curve (realized for flotation conditions) can be based on acceptance of the value of β and fitting the value of ϑ to it on the basis of balance equation by certain concentrate yield. This selection is not ambiguous; in flotation conditions the nature of the ore (feed) decides about it.

The analogical graph can be obtained for gravitational beneficiation through densimetric analysis. In this case the obtained values of β and ϑ are much more unequivocal because of the preciseness of separation in heavy liquid.

Figs. 5a, b, c present the relations between the sum of recoveries reduced by 100 from the copper recovery in concentrate, copper contents in concentrate β and copper contents in tailings ϑ which were obtained for the example of fractional flotation. On the basis of these Figures is possible to determine approximated optimal values which were equal, respectively, to: β_{max} = 59%, ϑ_{max} = 1,9% and ε_{max} = 90%, by constant value of α. The optimal values should be treated as natural response of flotation to feed quality, which is represented by its copper contents (by certain mineralogical composition of copper sulfides), contents of additional components which indicate some flotation properties and way of feed preparation (depth of comminution). It is worthy to notice that the order of experimental points is in accordance to flotation time for recovery and reverse to contents β and ϑ.

Considering the process of fractional flotation we have to take into account the order of various

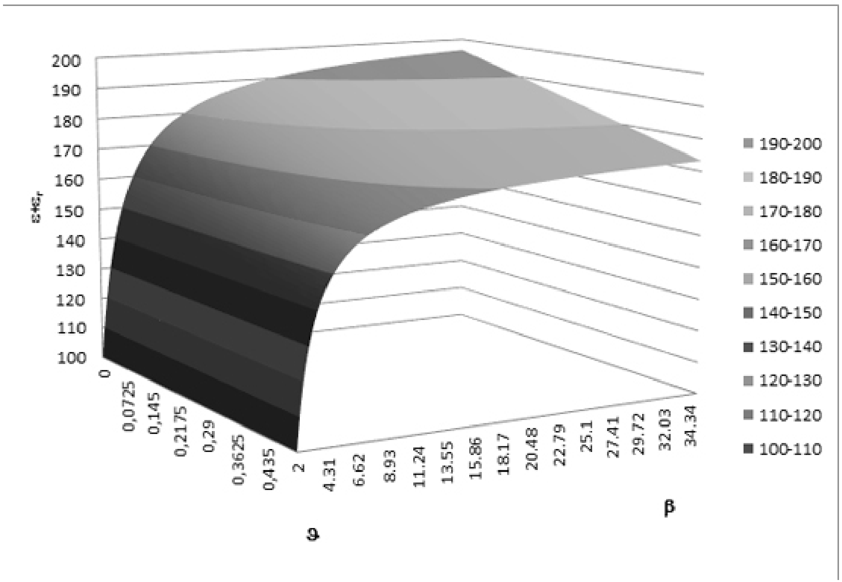


Fig. 4. Partition surface

Rys. 4. Powierzchnia rozdziału

ore fractions passing to concentrate which is related to the susceptibility of these fractions to being floated. Firstly, the pure minerals with minimal amount of outgrowths float. The yield of concentrate is rather low while the share of residuals in tailings is high. With the flotation time passing the particles having lower flotation abilities start to occur in concentrate so the residuals recovery ε_r becomes lower and lower (it occurs also from the structure of the equation).

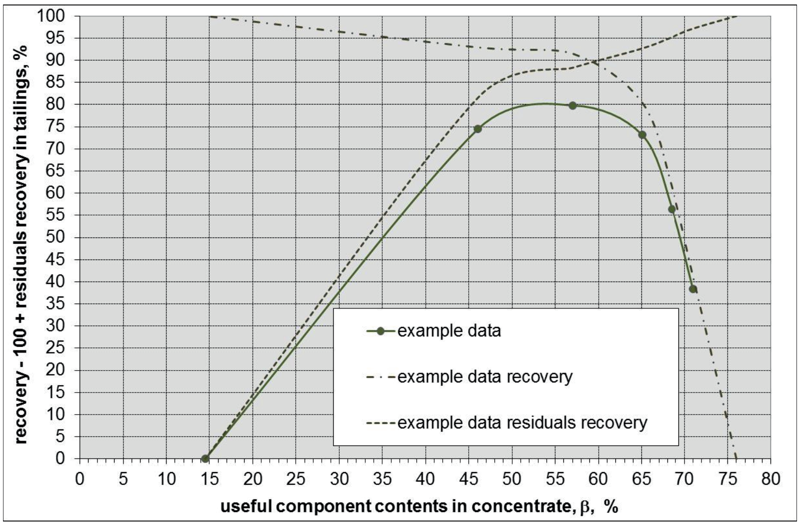
The application of sum of yields ($\varepsilon+\varepsilon_r$) leads also to determining the technological optimum. It is worthy to notice that the occurring of the variance of locations of intersection points between recoveries curves and optimal point is caused by the way of calculating concentrate and tailings yields for coal. The results of copper ore beneficiation were obtained as part of scientific work "Determination of influence of selection of particle size fraction below 5 mm from the first stage of grinding feed on classification and beneficiation processes in O/ZWR Region Polkowice". These investigations were carried out to determine both mineral liberation and its granulation through mineralogical investigations of the products, as well through determination of beneficiation characteristics by flotation conducted in laboratory conditions (scientific work O/ZWR, 2011; Foszcz, 2013).

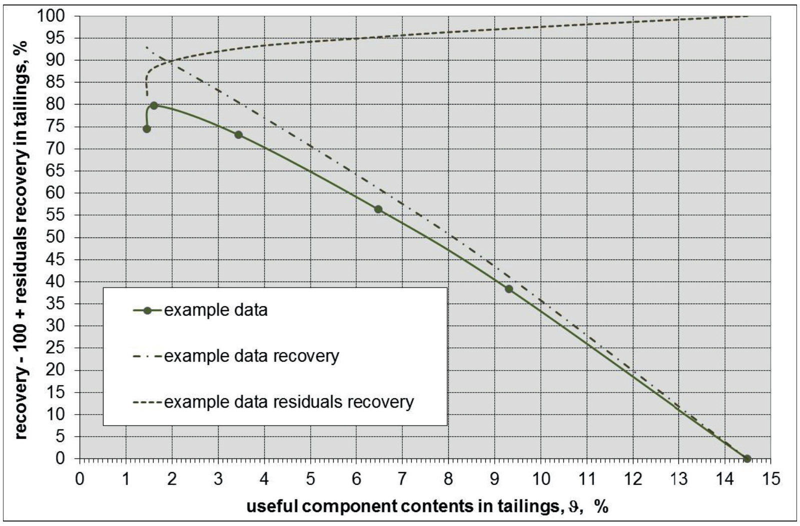
In purpose of presenting possibilities of considered methodology to evaluate beneficiation of raw materials the analyses for real laboratory results of ore originated from O/ZWR Region Polkowice and coal were conducted. The results are presented on Fig. 6.

Fig. 7 shows analogical to Fig. 5 set of curves describing course of coal separation in heavy liquids. The analyzed beneficiation results for coal were collected from research over coal usefulness to gasification process conducted in fluidized bed. It originated from Janina hard coal mine and its initial granulation was 0–20 mm (raw dust). According to Polish classification of coal types it was energetic coal. The material was divided into density fractions in homogenous heavy liquids (zinc chloride solutions). The detailed data concerning the researched coal were presented in the paper (Surowiak, 2013a).

Initial evaluation of the proposed methodology of determining technological optimum

The technological evaluation of mineral raw material quality is based on performing appropriate laboratory investigations which are direct applications of adequate beneficiation processes, like gravitational, flotation or magnetic ones. After initial preparation of the material (comminution, classification) the appropriate processes are conducted and their results are the basis for industrial results forecasting and technological systems designing. The mineral raw material characterizes with its mineral and petrographic construction and, dependably on its granulation, it would have potential possibilities of dividing into concentrate and tailings which can be characterized with appropriate contents of useful components. The ore





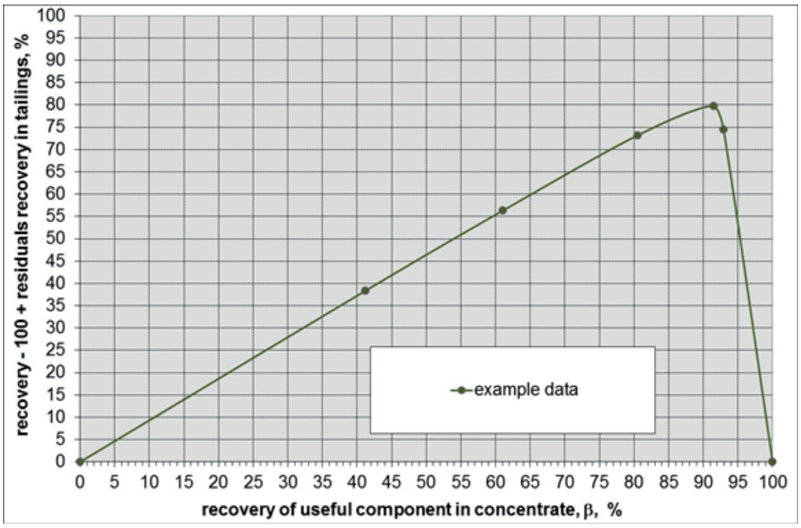


Fig. 5 Set of curves for ore beneficiation in system:
a – $\varepsilon=\varepsilon(\beta)$, $\varepsilon_r=\varepsilon_r(\beta)$ and $\varepsilon+\varepsilon_r-100=f(\beta)$
b – $\varepsilon=\varepsilon(\vartheta)$, $\varepsilon_r=\varepsilon_r(\vartheta)$ and $\varepsilon+\varepsilon_r-100=f(\vartheta)$
c – $\varepsilon+\varepsilon_r-100=f(\varepsilon)$

Rys. 5. Układ krzywych wzbogacalności rud:
a – $\varepsilon=\varepsilon(\beta)$, $\varepsilon_r=\varepsilon_r(\beta)$ and $\varepsilon+\varepsilon_r-100=f(\beta)$
b – $\varepsilon=\varepsilon(\vartheta)$, $\varepsilon_r=\varepsilon_r(\vartheta)$ and $\varepsilon+\varepsilon_r-100=f(\vartheta)$
c – $\varepsilon+\varepsilon_r-100=f(\varepsilon)$

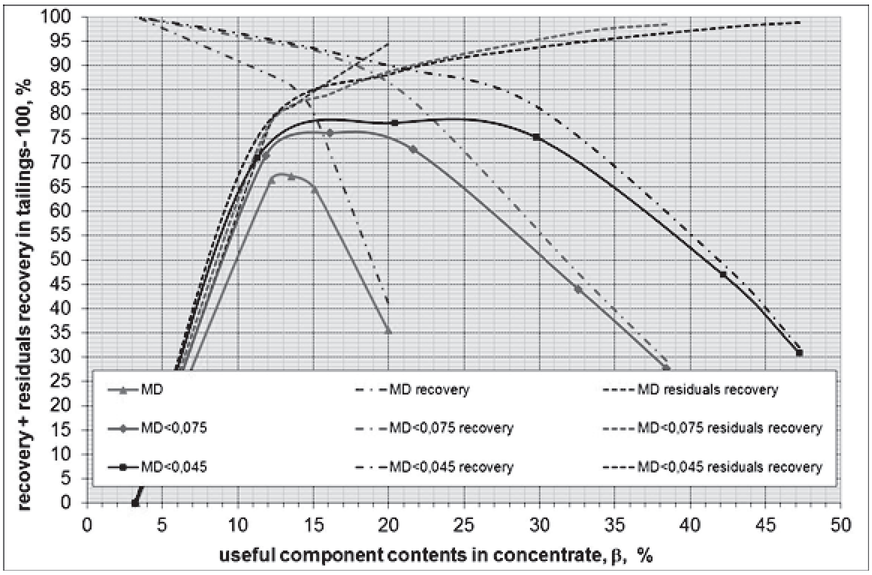


Fig. 6. Set of curves $\varepsilon = \varepsilon(\beta)$, $\varepsilon_r = \varepsilon_r(\beta)$ and $\varepsilon + \varepsilon_r\text{-}100 = f(\beta)$ for various courses of copper ore flotation. MD – feed for additional grinding mills, MD<0,075 – feed grinded below 0.075 mm, MD<0,045 – feed grinded below 0.045 mm.

Rys. 6. Układ krzywych $\varepsilon = \varepsilon(\beta)$, $\varepsilon_r = \varepsilon_r(\beta)$ and $\varepsilon + \varepsilon_r\text{-}100 = f(\beta)$ dla różnych przebiegów flotacji miedzi. MD – nadawa do układu domielania, MD<0,075 – nadawa rozdrabniona poniżej 0.075 mm, MD<0,045 – nadawa poniżej 0.045 mm

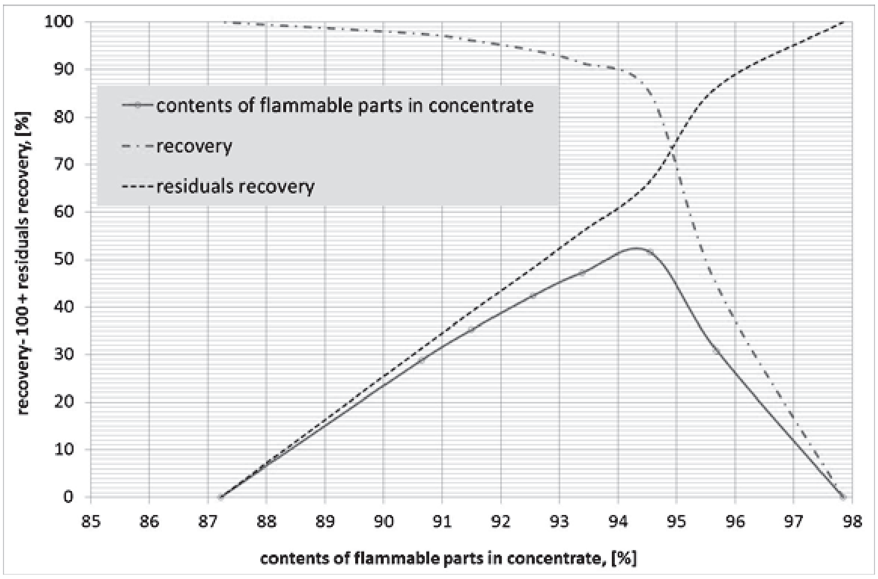


Fig. 7. Set of curves $\varepsilon = \varepsilon(\beta)$, $\varepsilon_r = \varepsilon_r(\beta)$ and $\varepsilon + \varepsilon_r\text{-}100 = f(\beta)$ for coal beneficiation.

Rys. 7. Układ krzywych $\varepsilon = \varepsilon(\beta)$, $\varepsilon_r = \varepsilon_r(\beta)$ and $\varepsilon + \varepsilon_r\text{-}100 = f(\beta)$ dla wzbogacania węgla

structure will also determine yields and recoveries of the component.

It can be accepted that for ore (material) prepared in certain way the natural, related to its construction, separation point exists which determines so-called technological optimum for it and for certain beneficiation method.

The methods of determining such optimum, discussed in this paper, were based on observed changes of frequency of useful component contents growth in the products. Such points were accepted as the points determining technological optimum – points of change of beneficiation quality type. The introduced term of technological optimum causes many additional issues connected with its uniqueness and possibilities of applications.

One of the basic questions is the problem of uniqueness of determination of technological optimum on the basis of various curves, which were in fact various types of experimental data presentation. This problem requires further investigation.

Furthermore, it occurred that the selection of the optimal point in method of sum of recoveries is also ambiguous. Fig. 6 presents the examples of three sets of curves connected with beneficiation of three samples of copper ore, prepared to flotation in various way. As it is easy to notice, the determination of the location of the curves peak for the cases of fine-grained feeds is hard. Additionally, the courses of graphs are charged with imprecision related to their automatic smoothing used by computer program. It seems to be correct to say that flotation is a random process which causes fuzzy effect around the peak of summarized curve and there is need of additional consideration of range concept of evaluation of potential ore abilities to beneficiation.

It is worthy to notice that indication of technological optimum for researched material is at the same time evaluation of its susceptibility to beneficiation. If the obtained levels of contents and recoveries would be not satisfying from the technological point of view then the planned technology should be changed by introduction of technological returns of material to the process, additional grinding, classification and further stages of flotation. It requires also supplementary investigations concerning mineralogical and petrographic composition of the material.

The conducted analysis of searching for technological optimum for the beneficiation results of two various materials (copper and coal) showed the universality of the proposed methodology of its determination. The proposed method gives also the possibilities of evaluating results of beneficiation by certain method or for certain technological system (way) of preparing of the feed to beneficiation process.

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Próba określenia wartości optymalnych wskaźników wzbogacania surowców mineralnych

W artykule przedstawiono techniki określenia tzw. optymalnych warunków wzbogacania surowców mineralnych. Jako materiał do doświadczeń została wybrana ruda miedzi, pochodząca ze złóż KGHM Polska Miedź S.A., rejon Polkowice oraz jeden z węgli kamiennych Górnego Śląska, pochodzący z ZG Janina. Stwierdzono, że wychodząc z równania bilansu możliwe jest ustalenie optymalnych charakterystyk wzbogacanego materiału. Do wyznaczenia optymalnego punktu, autorzy zastosowali powszechnie stosowane krzywe wzbogacalności, tj. krzywą Halbicha, krzywą Fuerstenaua oraz krzywą Madeja. Dla wszystkich zastosowano kryteria wyboru punktu optymalnego, za który uznano punkt największej krzywizny. Okazało się jednak, że dla zależności sum uzysku w koncentracie i uzysku reszt w odpadach możliwe jest wyznaczenie ekstremum, które można potraktować jako optimum technologiczne. Przyjmując, że α = const mamy w tym przypadku do czynienia z problemem wyznaczania ekstremum warunkowego funkcji sum uzysku składnika użytecznego w koncentracie oraz uzysku reszt w odpadzie o dwóch zmiennych β i ϑ. Metodykę tą zastosowano dla wybranej rudy miedzi oraz wybranego węgla kamiennego. Okazało się jednak, że wybór punktu optymalnego w metodzie sumy uzysków również nie jest jednoznaczny. Należy przy tym zwrócić uwagę, że wskazanie optimum technologicznego badanego surowca jest zarazem oceną jego podatności na wzbogacanie. Całość pracy zakończono wnioskami.

Słowa kluczowe: wskaźniki wzbogacania surowców, krzywa Halbicha, krzywa Fuerstenau, krzywa Madeja