Application of Variance Analysis to Compare Characteristics of Various Types of Hard Coal

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Abstract

In Polish nomenclature many types and subtypes of coal can be found which differ between themselves by individual characteristics. However, it is often that is no easy to recognize them properly on the basis on, for example, chosen numerical data describing their features. In the paper, the variance analysis was used as the tool of comparing analysis for three chosen types of coal which were collected from three various hard coal mines located in Upper Silesia. There were coals of type 31, 34.2 and 35. Each of coals was first screened and then additionally divided into density fractions by means of zinc chloride aqueous solution. Such prepared material was then investigated because of several chosen features, like combustion heat, ash contents, sulfur contents, volatile parts contents and moisture. Together with mass it gave seven–dimensional vector describing each of chosen fractions for all three types of coals. Then, the full variance analysis was conducted with investigation of all assumptions required to its conduction. The results served to elaborate conclusions.

Keywords: hard coal, coal type, multidimensional statistical analysis, variance analysis, particle fractions

Introduction

The grained materials can be described by means of many features. Usually, the basic ones are particle size and its density, but for example in case of coal many other important ones can be found which influence on differentiation of coal types. Many papers in literature concern typology of coals, but rarely are supported with detailed statistical analysis. Recently, with development of information science, multidimensional statistical analyzes gained significant meaning. It is worthy to mention here multidimensional visualization methods which can be treated as modern analysis tools. Among such types of methods the observational tunnels method can be found [Jamróz and Niedoba, 2014; Niedoba 2013a], parallel axes method [Niedoba and Jamróz, 2013]], Kohonen maps [Jamróz and Niedoba, 2015a], relevance maps [Niedoba, 2015], Principal Component Method [Niedoba, 2014], multidimensional scaling [Jamróz, 2014a] or autoassociative neural networks [Jamróz, 2014b]. In most of cases these methods were used to identify type of coal on the basis of measuring data. For example, it was used to define coal usefulness to gasification process [Marciniak–Kowalska et al., 2014]. Comparison of efficiency of these methods can be found in [Jamróz and Niedoba, 2015b]. Furthermore, application of complex statistical analysis can be also found in [Brożek et al., 2015; Niedoba, 2013b; Niedoba and Jamróz, 2013; Surowiak, 2014; Surowiak and Brożek, 2014a; 2014b; 2016]. In this paper the instrument was analysis of variance which assumptions can be found in [Krysicki et al., 2012; Tumidajski and Saramak, 2009]. In mineral processing it was used to evaluate flotation process [Agnew et al., 1995; Xiao and Vien, 2003] and recently also to recognize products of separation in fine coal jigs [Pięta, 2015].

The basic point of start in this paper is acceptance that grained material is characterized with multidimensional random variable $W = [w_1, w_2, \ldots, w_n]$, where w_i ($i = 1, \ldots, n$) are researched material properties. For various types of coals the values of individual w_i are significantly different. The purpose of the work is analysis of three types of coal properties (energetic coal, semi-coking coal and coking coal) and determination which of the investigated features allow to identify the type of coal. In papers [Jamróz and Niedoba, 2014; Marciniak–Kowalska et al., 2014] the identification of coal types by means of observational tunnels method was performed. On the basis of this analysis it was stated that three coal features, which were moisture, sulfur contents and volatile parts contents, were sufficient to identify the type of coal correctly.

Experiment

In the paper, the comparison of properties for three types of coals was done by means of statis-

Tab. 1. Ash contents for various types of coal

Tab. 1. Zawartość popiołu dla różnych typów węgli

Particle size d [mm]	Type 31		Type 34.2		Type 35	
	$\overline{w}$	s^2	$\overline{w}$	s^2	$\overline{w}$	s^2
0.50–1.00	20.55	236.74	21.14	234.96	24.55	238.62
1.00–3.15	23.59	259.55	21.78	211.38	24.55	238.62
3.15–6.30	23.52	259.45	20.00	150.33	26.01	247.78
6.30–8.00	28.00	230.16	22.14	222.41	27.73	275.05
8.00–10.00	27.23	174.36	23.77	221.34	27.04	287.50
10.00–12.50	34.32	485.70	22.34	199.54	28.10	276.59
12.50–14.00	27.12	174.23	22.31	166.24	27.72	277.26
14.00–16.00	27.76	192.25	21.81	273.92	27.02	273.92
16.00–20.00	25.55	171.46	28.77	585.76	27.94	300.58

Tab. 2. Results of Hartley's and Cochran's test for ash contents

Tab. 2. Wyniki testów Hartleya i Cochrana dla zawartości popiołu

d [mm]	*H* – Hartley's test	*G* – Cochran's test
0.50–1.00	1.02	0.33
1.00–3.15	1.79	0.39
3.15–6.30	1.09	0.37
6.30–8.00	1.24	0.40
8.00–10.00	1.65	0.42
10.00–12.50	2.04	0.50
12.50–14.00	1.67	0.44
14.00–16.00	1.46	0.33
16.00–20.00	3.42	0.54

tical methods, such as analysis of variance and verification of hypotheses about mean value and variance. These coals were marked by numbers 31, 34.2 and 35, according to Polish nomenclature of coal types. To this purpose, coals were initially sieved into particle size fractions –1.00, –3.15, –6.30, –8.00, –10.00, –12.50, –14.00, –16.00 and –20.00 mm and then into density fractions by means of aqueous zinc chloride solution (1.30, 1.40, 1.50, 1.60, 1.70, 1.80 and 1.90 g/cm^3). Each obtained size–density fraction was then investigated for such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents, analytic moisture. All these features, together with mass of size–density fraction gave seven various features for each type of coal.

In the paper the assumption was made that for each type of coal, material being the part of the same particle–size fraction which is material of particle size *d* and density ρ is one measuring object for which ash contents, sulfur contents and moisture were measured. In this way the following projection was determined:

$$(d, \rho) \rightarrow (w_1, w_2, w_3) \qquad (1)$$

where d_i is particle size in i^{th} fraction, ρ_i – particle density in ith fraction, w_1– ash contents, w_2 – sulfur contents, w_3 – moisture.

In such way the set of points (ρj, $w_1 j$, $w_2 j$, $w_3 j$), $j = 1,\ldots, m$ was obtained.

Ash contents

As the first feature ash contents in coal was selected to the analysis. On the basis of conducted laboratory researches the results were obtained, which were presented in Table 1,

where:

$\overline{w_{11}} = \frac{1}{n}\Sigma_{j=1}^{k} w_{11j}$ – is mean ash contents for coal, type 31 (2)

$\overline{w_{12}} = \frac{1}{n}\Sigma_{j=1}^{k} w_{12j}$ – is mean ash contents for coal, type 34.2 (3)

$\overline{w_{13}} = \frac{1}{n}\Sigma_{j=1}^{k} w_{13j}$ – is mean ash contents for coal, type 35 (4)

$s_i^2 = \frac{1}{n}\Sigma_{j=1}^{k}(w_{ij} - \overline{w_i})^2$ for i = 1, 2, 3 are variances for individual types of coals (5)

To check if ash contents differs significantly for individual types of coals the verification of hy-

Tab. 3. Results of F test for ash contents

Tab. 3. Wyniki testu F dla zawartości popiołu

***d* [mm]**	***F***
0.50–1.00	0.1182
1.00–3.15	0.0581
3.15–6.30	0.0423
6.30–8.00	0.2800
8.00–10.00	0.0990
10.00–12.50	0.6718
12.50–14.00	0.2562
14.00–16.00	0.2542
16.00–20.00	0.4122

Tab. 4. Sulfur contents for various types of coal

Tab. 4. Zawartość siarki dla różnych typów węgli

Particle size *d* [mm]	**Type 31**		**Type 34.2**		**Type 35**	
	[illegible]	s^2	[illegible]	s^2	[illegible]	s^2
0.50–1.00	1.18	0.15	0.72	0.04	0.60	0.01
1.00–3.15	1.41	0.26	0.62	0.06	0.59	0.02
3.15–6.30	1.23	0.21	0.67	0.05	0.60	0.01
6.30–8.00	1.30	0.23	0.63	0.04	0.62	0.02
8.00–10.00	0.65	0.08	0.55	0.04	0.94	0.03
10.00–12.50	0.94	0.06	0.38	0.01	0.53	0.03
12.50–14.00	1.20	0.27	0.39	0.07	0.62	0.15
14.00–16.00	0.87	0.19	0.72	0.26	0.56	0.03
16.00–20.00	1.04	0.27	0.36	0.09	0.79	0.13

where $\overline{w_{21}}$ are mean sulfur contents for individual types of (i=1, 2, 3) and s_i^2 are variances

pothesis H0: $\overline{w_{11}} = \overline{w_{12}} = \overline{w_{13}}$ (of equality of mean ash contents in individual types of coals) was performed for following fractions. To apply the analysis of variance, first the hypothesis of variance equality H0: $s_1^2 = s_2^2 = s_3^2$ must be verified.

To this purpose, the Hartley's test H and Cochran's test G were applied, where:

$$H = \frac{\max(s_1^2, s_2^2, s_3^2)}{\min(s_1^2, s_2^2, s_3^2)} \quad (6)$$

$$G = \frac{\max(s_1^2, s_2^2, s_3^2)}{s_1^2 + s_2^2 + s_3^2} \quad (7)$$

The results were presented in Table 2.

For the significance level α = 0.05 the critical range for Hartley's test is range (6.94, +∞) and for Cochran's test the range (0.73, +∞). Because all results of conducted tests are outside of the critical range so the analysis of variance can be applied.

To verify the hypothesis of mean value the F–Snedecor test was applied [Dobosz, 2001; Krysicki et al., 2012; Tumidajski, 1997] of form:

$F = \frac{(n-k)Q_G}{(k-1)Q_R}$ of Fisher–Snedecor distribution with k–1, n–k of freedom degrees (8)

where:

$$Q_G = \Sigma_{i=1}^{k} (\overline{w_i} - \overline{w})^2 \cdot n_i \quad (9)$$

$$Q_R = \Sigma_{i=1}^{k} \Sigma_{j=1}^{n_i} (w_{ij} - \overline{w_i})^2 \quad (10)$$

where $\overline{w} = \frac{1}{k} \overline{w_i}$

The results of F test were presented in Table 3.

The critical range for significance level α = 0.05 is range (3.97, +∞). Because all values of F test are outside the critical range so the hypothesis of equality of mean ash contents for considered types of coals can be accepted for each fraction.

Sulfur contents

Next, the similar analysis was performer for sulfur contents in coal. Data was presented in Table 4.

Also in this case the hypotheses of equality of mean values H0: $\overline{w_{21}} = \overline{w_{22}} = \overline{w_{23}}$ were checked. However, first the verification of equality of variances by means of Hartley's and Cochran's tests was done. The results of these tests were presented in Table 5.

Analyzing the results of tests is visible that starting from third fraction at least one test (by

Tab. 5. Results of Hartley's and Cochran's tests for sulfur contents

Tab. 5. Wyniki testów Hartleya i Cochrana dla zawartości siarki

d **[mm]**	***H*** **– Hartley's test**	***G*** **– Cochran's test**
0.50–1.00	9.35	0.75
1.00–3.15	14.77	0.76
3.15–6.30	8.33	0.64
6.30–8.00	11.95	0.70
8.00–10.00	2.69	0.54
10.00–12.50	6.44	0.62
12.50–14.00	3.61	0.55
14.00–16.00	7.64	0.54
16.00–20.00	3.04	0.55

Tab. 6. Results of *F* test for sulfur contents (starting from third fraction)

Tab. 6. Wyniki testu *F* dla zawartości siarki (zaczynając od trzeciej klasy ziarnowej)

d **[mm]**	***F***
3.15–6.30	6.30
6.30–8.00	5.04
8.00–10.00	4.92
10.00–12.50	16.27
12.50–14.00	6.38
14.00–16.00	0.89
16.00–20.00	4.41

Tab. 7. Results of Cochran–Cox test for sulfur contents

Tab. 7. Wyniki testu Cochrana–Coxa dla zawartości siarki

d **[mm]**	**H0: w((() = w(())**	**H0: w((() = w(())**	**H0: w((() = w(())**
0.50–1.00	2.60	3.40	1.39
1.00–3.15	3.38	3.79	0.26

significance test α = 0.05) allow to accept that variances for individual types of coals are equal and for these fractions the *F* test was performed, which results were presented in Table 6.

In all cases, except fraction (14.00, 16.00) the test result for significance level α = 0.05 is within the critical range, which is (3.97, +∞). That means that hypothesis about mean sulfur contents in individual fractions for various coal types should be rejected (except fraction (14.00, 16.00). Because for fractions (0.50–1.00) and (1.00, 3.15) is not possible to apply *F* test, the Cochran–Cox test was used instead [Dobosz, 2001; Krysicki et al., 2012; Tumidajski, 1997] of equality of two mean values. This test is conducted according to the formula

$$C = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}} \quad (11)$$

The hypotheses $H_0: \overline{w_{21}} = \overline{w_{22}}$; $H_0: \overline{w_{21}} = \overline{w_{23}}$ and $H_0: \overline{w_{22}} = \overline{w_{23}}$ were verified. For significance level α = 0.05 the critical range of C test is (–∞, –1.94)∨(1.94, +∞).

Results of Cochran–Cox test were presented in Table 7.

From the obtained results is possible to conclude that only hypothesis $H_0: \overline{w_{22}} = \overline{w_{23}}$ of equality of mean sulfur contents for coal, types 34.2 and 35 was verified correctly for both fractions.

Moisture

The next investigated coal feature is material's moisture, which results were given in Table 8.

As in previous cases, first the possibility of applying analysis of variance was verified by checking hypothesis of equality of variances by means of Hartley's and Cochran's tests. The results of both tests were presented in Table 9.

All values of test, apart from the fraction (12.50; 14.00) and (14.00; 16.00) were within the critical range of the test so is not possible to apply *F* test. However, this test was used for fractions (12.50; 14.00) and (14.00; 16.00)

Tab. 8. Moisture for various types of coals

Tab. 8. Wilgotność dla różnych typów węgli

Particle size *d* **[mm]**	**Type 31**		**Type 34.2**		**Type 35**	
	$\overline{w}$	s^2	$\overline{w}$	s^2	$\overline{w}$	s^2
0.50–1.00	3.63	0.81	0.68	0.01	1.08	0.01
1.00–3.15	3.02	0.61	0.68	0.03	1.02	0.001
3.15–6.30	3.00	0.38	0.99	0.03	1.36	0.03
6.30–8.00	3.00	0.22	1.04	0.01	1.22	0.02
8.00–10.00	2.92	0.40	1.16	0.02	1.36	0.03
10.00–12.50	2.44	0.90	1.12	0.01	1.38	0.01
12.50–14.00	2.95	0.41	1.01	0.22	1.41	0.01
14.00–16.00	3.04	0.09	0.88	0.04	1.34	0.01
16.00–20.00	3.30	0.57	1.18	0.01	1.41	0.03

where $\overline{w_{3k}}$ is mean moisture for k^{th} coal type and s_k^2 is variance

Tab. 9. Results of Hartley's and Cochran's tests for moisture

Tab. 9. Wyniki testów Hartleya i Cochrana dla wilgotności

d **[mm]**	***H* – Hartley's test**	***G* – Cochran's test**
0.50–1.00	0.50	0.97
1.00–3.15	89.90	0.95
3.15–6.30	14.73	0.87
6.30–8.00	20.47	0.87
8.00–10.00	15.97	0.89
10.00–12.50	97.57	0.97
12.50–14.00	45.06	0.64
14.00–16.00	5.80	0.63
16.00–20.00	67.90	0.93

Tab. 10. Results of F test for moisture (fractions (12.50, 14.00) and (14.00, 16.00))

Tab. 10. Wyniki testu F dla wilgotności (klasy (12.50, 14.00) oraz (14.00, 16.00))

d **[mm]**	***F***
12.50–14.00	29.52
14.00–16.00	66.21

Tab. 11. Results of Cochran–Cox test for moisture

Tab. 11. Wyniki testu Cochrana–Coxa dla wilgotności

d **[mm]**	**H_0: $\overline{w} = \overline{w}$**	**H_0: $\overline{w} = \overline{w}$**	**H_0: $\overline{w} = \overline{w}$**
0.50–1.00	8.01	6.91	6.96
1.00–3.15	7.18	6.27	6.49
3.15–6.30	7.71	6.24	3.75
6.30–8.00	9.82	8.81	0.90
8.00–10.00	6.66	5.82	2.16
10.00–12.50	4.97	2.73	4.20
16.00–20.00	6.82	5.96	2.78

and the obtained results were presented in Table 10.

Because on significance level $\alpha = 0.05$ the test results for both fractions are within critical range which was the range (3.97, +∞), so the hypothesis of equality of mean moisture in various coal types should be rejected.

For the remaining fractions the Cochran–Cox test of equality of two mean values was used. The results were presented in Table 11.

Only for fraction (6.30; 8.00) in cases of coal types 34.2 and 35 the result of test of equality of moisture lied outsider the critical range for significance level $\alpha = 0.05$, which was the range (1.94,

+∞). The other results contain within this range. So, it can be stated that the hypothesis of equality of mean moisture for various coal types should be rejected.

Conclusions

Analyzing the obtained results it can be stated that the results of moisture and sulfur contents are necessary to identify the type of coal, while ash contents does not differentiate significantly the coal type. However, because of the fact that for some fractions hypotheses of equality between mean sulfur contents and mean moisture were not rejected, in some cases another coal feature should be used, like volatile parts contents. The obtained results were completely in accordance with the results obtained by means of observational tunnels method, presented in [6, 11].

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Literatura – References

1. AGNEW R., STRATTON−CRAWLEY R., and WILSON S.W. „Evaluation of flotation performance using variance spectrum analysis." *Minerals Engineering* 8/1–2(1995): 51–62.
2. BROŻEK M., SUROWIAK A., and JAROSIŃSKI A. „Beneficiation of chromium waste by means of magnetic and gravitational separation." *Archives of Metallurgy and Materials* 60/3(2015): 1737–1744.
3. DOBOSZ M. *Statistical analysis of research results.* Warszawa: Akademicka Oficyna Wydawnicza, 2001.
4. JAMRÓZ D. „Application of multidimensional scaling to classification of various types of coal." *Archives of Mining Sciences* 59/2(2014): 413–425.
5. JAMRÓZ D. „Application of multi–parameter data visualization by means of autoassociative neural networks to evaluate classification possibilities of various coal types." *Physicochemical Problems of Mineral Processing* 50/2(2014): 719–734.
6. JAMRÓZ D., NIEDOBA T. „Application of the observational tunnels method to select a set of features sufficient to identify a type of coal." *Physicochemical Problems of Mineral Processing* 50/1(2014): 185–202.
7. JAMRÓZ D., NIEDOBA T. „Application of multidimensional data visualization by means of self–organizing Kohonen maps to evaluate classification possibilities of various coal types." *Archives of Mining Sciences* 60/1(2015): 39–50.
8. JAMRÓZ D., NIEDOBA T. „Comparison of selected methods of multi–parameter data visualization used for classification of coals." *Physicochemical Problems of Mineral Processing* 51/2(2015): 769–784.
9. KRYSICKI J., BARTOS D., DYCZKA W., KRÓLIKOWSKA K., WASILEWSKI M. *Probability and mathematical statistics in examples*, part II, Warszaw: PWN, 2012.
10. MARCINIAK−KOWALSKA J., NIEDOBA T., SUROWIAK A., TUMIDAJSKI T. „Multi–criteria evaluation of coal properties in terms of gasification." *Archives of Mining Sciences* 59/(2014): 677–690.
11. NIEDOBA T. *Multidimensional characteristics of random variables in description of grained materials and their separation processes.* Kraków: Wydawnictwo Instytutu Gospodarki Surowcami Mineralnymi i Energią PAN, Studia, Rozprawy, Monografie, vol. 182, 2013.
12. NIEDOBA T. „Methodological elements of applying two- and multidimensional distributions of grained materials properties to coal benefication." *Mineral Resources Management* 29/2(2013): 155–172.
13. NIEDOBA T. „Multi–parameter data visualization by means of principal component analysis (PCA) in qualitative evaluation of various coal types." *Physicochemical Problems of Mineral Processing* 50/2(2014): 575–589.
14. NIEDOBA T. „Application of relevance maps in multidimensional classification of coal types." *Archives of Mining Sciences* 60/1(2015): 93–106.
15. NIEDOBA T., JAMRÓZ D. „Visualization of multidimensional data in purpose of qualitative classification of various types of coal." *Archives of Mining Sciences* 58/4(2013): 1317–1331.
16. NOWAK A., SUROWIAK A. „Methodology of the efficiency factors of fine grained clayish suspensions separation in multileveled hydrocyclone systems." *Archives of Mining Sciences* 58/4(2013): 1209–1220.
17. PIĘTA P. „One–dimensional and two–dimensional analyses of hard coal separation in a jig." *Journal of the Polish Mineral Engineering Society* 35/1(2015): 133–138.

18. SUROWIAK A. „Influence of particle density distributions of their settling velocity for narrow size fractions." *Mineral Resources Management* 30/1(2014): 105–122.
19. SUROWIAK A., BROŻEK M. „Methodology of calculation the terminal settling velocity distribution of irregular particles for values of the Reynold's number." *Archives of Mining Sciences* 59/2(2014): 553–562.
20. SUROWIAK A., BROŻEK M. „Methodology of calculation the terminal settling velocity distribution of spherical particles for high values of the Reynold's number." *Archives of Mining Sciences* 59/1(2014): 269–282.
21. SUROWIAK A., BROŻEK M. „A physical model of separation process by means of JIGS." *Physicochemical Problems of Mineral Processing* 52/1(2016): 228–243.
22. TUMIDAJSKI T. *Stochastic analysis of grained materials characteristics and their separation processes*, Kraków: Wydawnictwo AGH, 1997.
23. TUMIDAJSKI T., Saramak D. *Methods and models of mathematical statistics in mineral processing*, Kraków: Wydawnictwo AGH, 2009.
24. XIAO Z., VIEN A. „Limitations of variance analysis using propagation of variance." *Minerals Engineering* 16/5(2003): 455–462.

Zastosowanie analizy wariancji do porównania charakterystyk różnego typu węgli kamiennych

W polskiej nomenklaturze istnieje wiele typów i podtypów węgla, które różnią się między sobą różnymi cechami. Jednakże, często nie jest łatwo rozpoznać je na podstawie, na przykład, wybranej zmiennej numerycznej opisującej ich cechy. W artykule zastosowano analizę wariancji jako narzędzia porównawczego dla trzech typów węgli kamiennych, które zostały pobrane z trzech kopalni zlokalizowanych na Górnym Śląsku. Były to węgle typów 31, 34.2 oraz 35. Każdy z węgli został najpierw przesiany a następnie dodatkowo rozdzielony na frakcje gęstościowe przy użyciu wodnego roztworu chlorku cynku. Tak przygotowany materiał został następnie zbadany ze względu na kilka wybranych cech, takich jak ciepło spalania, zawartość popiołu, zawartość siarki, zawartość części lotnych oraz wilgotność. Wraz z masą dało to siedmiowymiarowy wektor opisujący każdą z wybranych frakcji dla wszystkich trzech typów węgli. Następnie, przeprowadzono pełną analizę wariancji z badaniem wszystkich założeń wymaganych do jej przeprowadzenia. Wyniki posłużyły do opracowania wniosków.

Słowa klucze: węgiel kamienny, typ węgla, wielowymiarowa analiza statystyczna, analiza wariancji, klasy ziarnowe