



# Estimation of a Horizontal Refraction in a Geodetic Network

Marek BAJTALA<sup>1)</sup>, Štefán SOKOL<sup>2)</sup>, Pavel ČERNOTA<sup>3)</sup>

<sup>1)</sup> Ing., Ph.D.; Department of Surveying, faculty of Civil Engineering, STU – Slovak University of Technology in Bratislava, radlinského 11, 85105 Bratislava, Slovak Republic; email: marek.bajtala@stuba.sk, tel.: +421 2 59274392

<sup>2)</sup> Prof. Ing., Ph.D.; Department of Surveying, faculty of Civil Engineering, STU – Slovak University of Technology in Bratislava, radlinského 11, 85105 Bratislava, Slovak Republic; email: stefan.sokol@stuba.sk, tel.: +421 2 59274689

<sup>3)</sup> Doc. Ing., Ph.D.; Institute of Geodesy and Mine Surveying, Faculty of Mining and Geology, VŠB – Technical University of Ostrava, 17. Listopadu 15/2172, 70833 Ostrava – Poruba, Czech Republic; email:pavel.cernota@vsb.cz, tel.: +420 597 321 234

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## Abstract

The proposed thesis is focused on reviewing the horizontal refraction during measuring in local geodetic networks. Based on elaborated theoretical assumptions the refraction model was arranged in a way that the questions were solved by the identity of the horizontal refraction. The necessary part of the application of the refraction model is the problem of optimizing the geodetic networks and deducting the method of the variance components, which play a major role in the precision of results. The independent acquisition of the thesis lies in the area of modelling the horizontal refraction with a stochastic approach, where the values of the refraction are gained through the measured data of the horizontal direction. In the model, it is necessary to identify the parameters according to the entire measurements and between two points. According to the attained results, I can state that the horizontal refraction was identified mainly on the measurements that were confirmed near the dam in most of the measured experiments. The experimented measurements were confirmed based on the assumptions that are in theoretical capitol, where the assumptions of the effect of the horizontal refraction are measured near the dam, river, shores, and valleys.

Keywords: horizontal refraction, submatrix of refraction components, coefficient of refraction identification, estimation of refraction angle

## Introduction

Realization of new, more demanding and more complex types of structures and constructions in engineering surveying requires, in terms of desired accuracy, high demands on the accuracy for establishing local control network. Currently produced total stations have reached high accuracy for measurement of horizontal directions, vertical angles as well as distances. The limiting factor of achieved accuracy is the impact of an environment. The negative impact of refraction on the estimation of control network parameters is given in (Gašincová, Gašinec, 2010), (Černota, Staňková, Brejcha, Bouček, 2013). Therefore, surveyors face a challenge to comprehensively understand the laws and phenomena of refraction. Due to this facts, the submitted article deals with this issue, specifically with estimation of the horizontal refraction in a control network.

The environment, in which surveying measurements are performed in local horizontal networks, is mostly heterogeneous, and thus light propagation is more difficult. When the light rays propagate from an instrument station to a target, they travel through the environment with different densities, bend and change their original direction. When the refractivity index changes gradually, then the path of a light ray gradually curves as well and the observer sights it as a tangent from

the point of observation. If we measure horizontal directions, these are affected by the horizontal refraction, i.e. bend of the beam in the horizontal direction. Horizontal directions are measured between the tangents to the sights rays connecting the endpoints. Due to the diversity of air the light rays bend and the results of measurement do not correspond to real values. Differences that arise are called observational errors from the impact of the horizontal refraction (bend) of the light ray.

## Possibilities of issue solving

There are several ways to reduce the impact of the refraction on the results of surveying measurements. In particular, these include execution of the angle measurements at the time of minimal refraction in the atmosphere surface layer. Sufficient reduction of the refraction impact can be achieved by optimal organization of observation, that the total impact of the refraction on each station or group of stations is excluded, or minimal.

Another way of determining the refraction is by means of additional measurements of parameters of the atmosphere surface layer. These approaches are more laborious, nevertheless they enable to obtain more objective information about the effect of the refraction.

Finally, a possible approach of determining the refraction is its estimation directly within the pro-

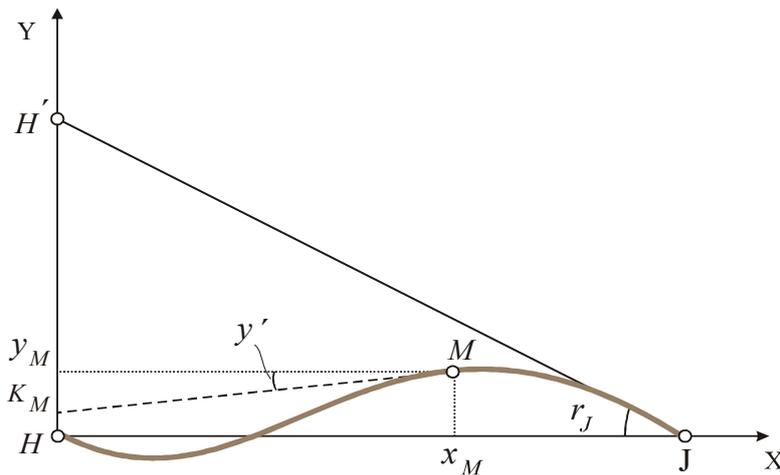


Fig. 1. Refraction angle

Rys. 1. Kąt załamania

cessing of the measurements in a control network. This method is the most suitable from the practical as well as economical view and is a subject of this article. Before we start with the development of a model for determining the refraction, it is necessary to find the basic laws of light ray bending in the atmosphere and to obtain knowledge about the course and shape of sight passing between two points of control network (Bajala, Sokol, Ježko 2007).

### Determination of a refraction angle

The basic measure of refraction is a refraction angle  $r_j$  between the tangent to the light ray and the straight line joining the source and receiver of light. The projection this beam on the horizontal plane is the angle of horizontal refraction  $r_{bocn}$ . To solve surveying tasks is sufficient to deal with research of small refraction angles (Junošev, 1969).

To define the equation for refraction angle at point J, we assume projection of ray  $HMJ$  on the plane passing through the light receiver J (fig. 1). The origin of the coordinate system is in the transmitter H and the X axis passes through point J (receiver). Angle  $r_j$  at point J between the tangent  $H'J$  and straight line  $HJI$  is the projection of the refraction angle on the plane XOY.

If we set point M to point J, then the tangent  $MK_M$  indicates section  $HH'$  on the Y axis, determined as the sum of elementary sections  $d\Delta$ , which can be written as integral:

$$HH' = \int_0^{x_j} d\Delta = - \int_0^{x_j} xy''_x dx \quad (1)$$

From right-angled triangle  $H'JH$  we can determine the unknown refraction angle  $r_j$ :

$$r_j^{cc} = \rho^{cc} \operatorname{tgr} r_j = - \frac{\rho^{cc}}{x_j} \int_0^{x_j} xy''_x dx. \quad (2)$$

If the refraction angles are small, we can write the resulting equation for the refraction angle  $r_j$  (Junošev, 1969):

$$r_j^{cc} = - \frac{\rho^{cc}}{x_j} \int_0^{x_j} \operatorname{grad}_{y,n} x dx. \quad (3)$$

In the study of the refraction in the horizontal plane, the OY axis is oriented parallel with the plane instrument's horizon and then for the horizontal refraction angle applies:

$$r_{bocn}^{cc} = - \frac{\rho^{cc}}{d} \int_0^D \operatorname{grad}_{bocn,n} x dx, \quad (4)$$

where  $d$  is horizontal projection of distance  $D$  and  $\operatorname{grad}_{bocn,n}$  is horizontal gradient of the refractivity index in direction perpendicular to the beam path (Junošev, 1969).

In the study of refraction in the atmosphere surface layer, it is important to allocate the impact of particular factors of refractivity index gradient on the value of refraction angle. Every, specifically taken part of the atmosphere affected by refraction factor we will characterise by the mean value of the refractivity index gradient  $g_i$  and if it does not change its direction to the contrary, we can consider  $\operatorname{grad}_{bocn,n}$  function to be constant. Thus, we can divide part of the beam path  $D$  into  $k$  sections. Parts of the atmosphere with the mean refractivity index gradient we can call quasi stationary blocks of refraction. The mean refractivity index gradient  $g_i$  is called vector of refraction. If along the entire distance  $D$  except section I horizontal gradient  $n$  equals  $g^0$  and on section I we call it as then:

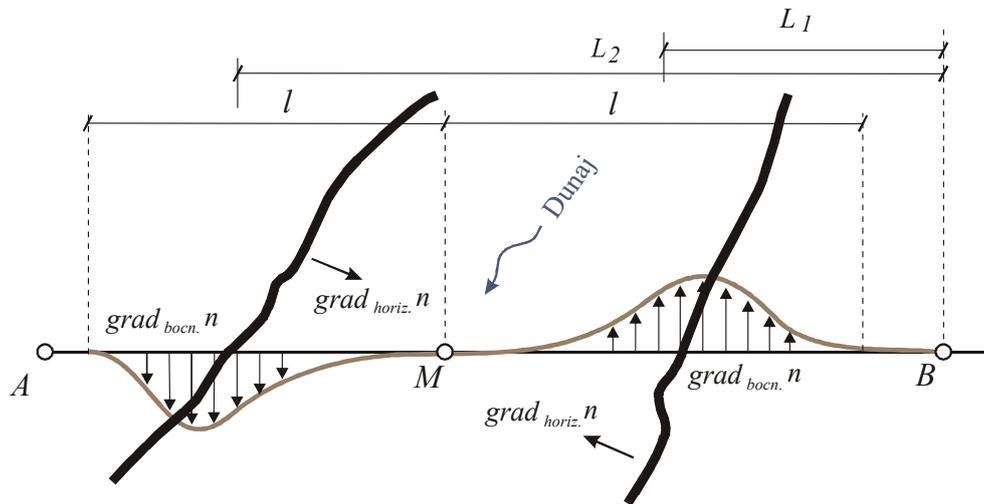


Fig. 2. Refraction blocks over water-level of a river  
 Rys. 2. Blokady załamania nad poziomem wody w rzece

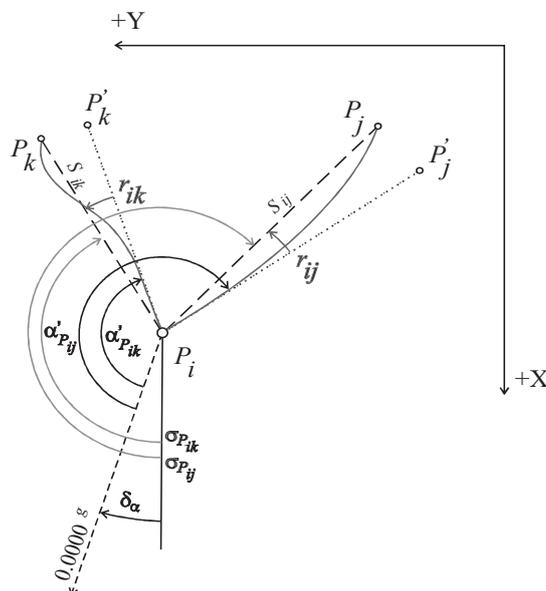


Fig. 3. Influence of horizontal refraction  
 Rys. 3. Wpływ załamania poziomego

$$r_{bocn.}^{cc} = -\rho^{cc} g^0 l \frac{L}{d}. \quad (5)$$

The value of refraction angle is proportional to the vector of refraction in the block, the length of block and the length coefficient. Therefore, the greatest influence on the refraction angle has the refractivity index gradient in the vicinity of an instrument.

### Typical cases of refraction blocks

The previous considerations were related to arbitrary distribution of sights rays relative to the refraction blocks. In practical surveying measurements there are cases of sight distribution depen-

dent on the local objects. The most common case is the division of network into two parts with different surfaces, which are formed by anomalous temperatures in the atmosphere surface layer, such as division by railways, highways, rivers, mountain valleys, city streets etc.

The above mentioned cases of division by valleys, rivers, city streets, we can generalize into a single idealized case. In this case, sights connect two points which are located in different atmospheric conditions and state of air between points is different from these surfaces. In this case directions of horizontal temperature gradients change into the opposite at some point of sight (in the vicinity of point). Accordingly, direc-

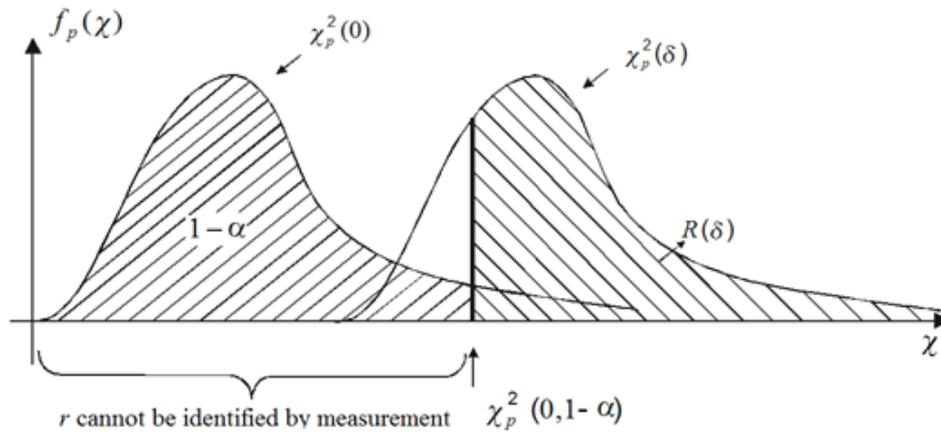


Fig. 4. Probability of identification none-zero refraction vector

Rys. 4. Prawdopodobieństwo identyfikacji zero-jedynkowego wektora załamania

tions of lateral temperature gradient of sight rays change.

It can be seen that refraction angles of straight sight and opposite sight are the same. Since the direction of angle at point A is on the left and at point B on the right (or vice versa), horizontal refraction will not affect the angles calculated from the measured directions. Effect of refraction will occur only in the direction of sight, which is parallel to the river bank (Bajtala, Sokol, Jeżko 2007).

#### Model of refraction in a control network

Processing of a control network is commonly executed by the least squares method (LSM), robust method or other modified estimation procedures (Weiss, 1996). In our case, we use standard approach using LSM. In the mathematical model of a horizontal control network, coordinates of given (fixed) points are considered as constant and coordinates of determined points  $\beta(y,x)$  are unknown. By realization of the observational plan, measured parameters  $Y(s,\alpha)$  (distance, directions or other measured variables) will be assigned to the horizontal control network.

$$\mathbf{Y}(s_{i,j}) = \sqrt{(Y_j - Y_i)^2 + (X_j - X_i)^2} + \varepsilon_s = \tilde{s}_{i,j} + \varepsilon_s \quad (6)$$

When measuring directions in a control network, it is necessary to consider, that we don't measure bearings but oriented directions, which have within one station an orientation shift  $\delta\alpha_i$ .

$$\mathbf{Y}(\sigma_{i,j}) = \arctg \frac{Y_j - Y_i}{X_j - X_i} + \delta\alpha_i + \varepsilon_\alpha = \tilde{\alpha}_{i,j} + \delta\alpha_i + \varepsilon_\alpha, \quad (7)$$

where  $\varepsilon_\alpha$  is accuracy of measurement of horizontal directions.

We define a stochastic model in which the vector of measured parameters can be written as follows:

$$\mathbf{Y} = \mathbf{f}_0 + \mathbf{F}\delta\beta + \mathbf{G}\delta\alpha + \varepsilon; \quad \text{Var}(\mathbf{Y}) = \Sigma, \quad (8)$$

where:

$$\mathbf{f}_0 = \mathbf{f}(\beta_0, \alpha_0)$$

$$\mathbf{F} = \partial \mathbf{f}(\beta) / \partial \beta^T |_{\beta=\beta_0}$$

$$\mathbf{G} = \partial \mathbf{f}(\alpha) / \partial \alpha^T |_{\alpha=\alpha_0}$$

Thus defined model of measurement in the control network we extend with the impact of horizontal refraction (vector  $r$ ) on the horizontal directions.

Vector of measured parameters can be written as:

$$\mathbf{Y} = \mathbf{f}_0 + \mathbf{F}\delta\beta + \mathbf{G}\delta\alpha + \mathbf{H}r + \varepsilon; \quad \text{Var}(\mathbf{Y}) = \Sigma, \quad (9)$$

where

$$\mathbf{f}_0 = \mathbf{f}(\beta_0, \alpha_0, r_0) \quad \text{and} \quad \mathbf{H} = \partial \mathbf{f}(r) / \partial r^T |_{r=r_0}$$

matrix is composed by elements of partial derivation of vector  $f_0$  according to approximate parameters of refraction angles  $r_0$ .

The model can further written as:

$$\mathbf{Y} = \mathbf{f}_0 + (\mathbf{F}, \mathbf{G}, \mathbf{H}) \begin{pmatrix} \delta\beta \\ \delta\alpha \\ r \end{pmatrix} + \varepsilon; \quad \text{Var}(\mathbf{Y}) = \Sigma. \quad (10)$$

Estimation of unknown parameters  $(\delta\beta^T, \delta\alpha^T, \hat{r}^T)^T$  can be written:

$$\begin{pmatrix} \delta\hat{\beta} \\ \delta\hat{\alpha} \\ \hat{r} \end{pmatrix} = \begin{bmatrix} \mathbf{F}^T \\ \mathbf{G}^T \\ \mathbf{H}^T \end{bmatrix} \Sigma^{-1} (\mathbf{F}, \mathbf{G}, \mathbf{H})^{-1} \begin{pmatrix} \mathbf{F}^T \\ \mathbf{G}^T \\ \mathbf{H}^T \end{pmatrix} \Sigma^{-1} (\mathbf{Y} - \mathbf{f}_0). \quad (11)$$

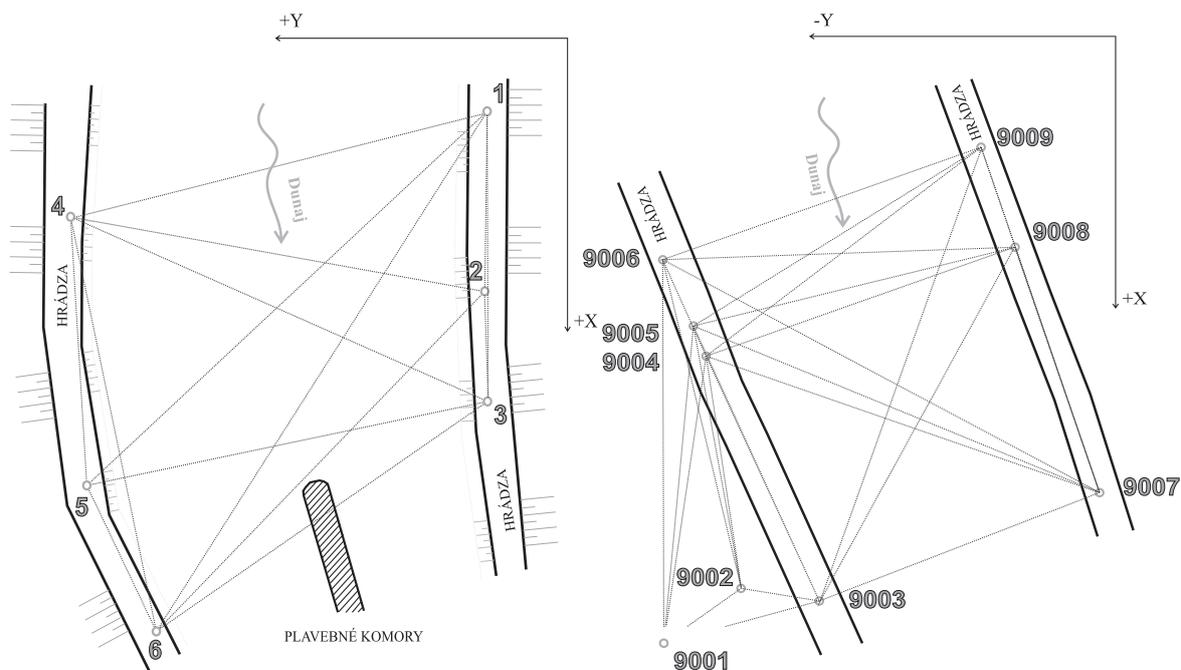


Fig. 5. a) Control network at Gabčíkovo water work, b) Control Network – Vojka nad Dunajom – Kyselica  
 Rys. 5. a) sieć kontrolna wody w Gabčikovie; b) sieć kontrolna w Vojce nad Dunajom – Kyselicy

Tab. 1. The resulting values of horizontal refraction and meteorological conditions: E1–E4

Tab. 1. Wyniki załamania poziomego i warunki meteorologiczne: E1–E4

Experiment	season	weather			Temp. [°C]	Press. [hPa]	Hum. [%]
E1	December	Cloudy, damp, moderate wind			2 - 3	983 - 984	96 - 98
Refractions	direction <b>r</b>	1 - 5	2 - 4	3 - 5	<i>Identified refraction effect in directions over the water surface</i>		
		-10 <sup>cc</sup>	7,6 <sup>cc</sup>	-8,3 <sup>cc</sup>			
E2	February	Partly cloudy, moderate wind			8 - 11	986 - 990	84 - 87
Refractions	direction <b>r</b>	4 - 6			<i>Identified influence of refraction of the directions of the dike</i>		
		-10,5 <sup>cc</sup>					
E3	March	Partly cloudy, moderate wind			10 - 14	1002 - 1005	74 - 77
Refractions	direction <b>r</b>	1 - 2	3 - 6	4 - 6	<i>Identified influence of refraction 2 directions for the dam and 1 direction over the water surface</i>		
		5,5 <sup>cc</sup>	-9,3 <sup>cc</sup>	-9,1 <sup>cc</sup>			
E4	April	Partly cloudy and windy			14 - 18	999 - 1000	80 - 82
Refractions	direction <b>r</b>	1 - 3	4 - 5	4 - 6	<i>Identified influence of refraction of the directions of the dike</i>		
		9,0 <sup>cc</sup>	-15,5 <sup>cc</sup>	-14,0 <sup>cc</sup>			

Resulting covariance matrix of unknown parameters  $(\delta\hat{\beta}^T, \delta\hat{\alpha}^T, \hat{r}^T)^T$  has following form:

$$\text{Var} \begin{pmatrix} \delta\hat{\beta} \\ \delta\hat{\alpha} \\ \hat{r} \end{pmatrix} = \begin{bmatrix} F^T \\ G^T \\ H^T \end{bmatrix} \Sigma^{-1}(F, G, H) \quad (12)$$

By means of created model we can analyse and assess the significance of the impact of horizontal refraction on the measurement of horizontal directions. In next section we divide the resulting covariance matrix in particu-

lar blocks, from which is the most important, in terms of further analysis, submatrix of refraction components:

$$\text{Var} \begin{pmatrix} \delta\hat{\beta} \\ \delta\hat{\alpha} \\ \hat{r} \end{pmatrix} = \begin{bmatrix} F^T \\ G^T \\ H^T \end{bmatrix} \Sigma^{-1}(F, G, H) = \begin{pmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{pmatrix} \quad (13)$$

where  $\text{var}(\hat{r}) = (33)$ .

From the mentioned structure of covariance matrix is interesting in terms of refraction appearance a submatrix (33) =

Tab. 2. Estimates of refraction – E5, E5S experiments

Tab. 2. Ocenę załamania – eksperymenty E5, E5S

Directions	Refractions E5, E5S		Standar deviation E5, E5S		Coefficient of identification E5,E5S $k_r$ critical = 3,84	
	r [°]	r [°]	$\sigma_r$ [°]	$\sigma_r$ [°]	kr	kr
1-2	5,2	6,3	2,6	2,3	4,0	7,2
1-3	9,9	10,7	2,6	2,4	14,6	20,8
1-4	-3,8	-4,0	2,6	2,4	2,2	2,7
1-5	4,7	3,1	2,7	2,5	3,1	1,5
1-6	4,4	3,6	2,7	2,6	2,8	2,0
2-3	-1,5	-3,7	2,6	2,3	0,4	2,7
2-4	-4,9	-4,6	2,6	2,4	3,5	3,6
2-6	-3,7	-4,0	2,7	2,5	1,9	2,6
3-4	-2,0	1,1	2,6	2,3	0,6	0,2
3-5	3,0	-0,7	2,6	2,4	1,3	0,1
3-6	-4,3	-4,2	2,6	2,4	2,7	3,0
4-5	-7,1	-6,0	2,6	2,3	7,5	6,6
4-6	-7,3	-11,0	2,6	2,4	8,1	21,7
5-6	-8,7	-8,5	2,6	2,3	11,6	13,7

$[H^T \Sigma^{-1} H] = \text{var}(\hat{r})$  (Bajala, Sokol, Ježko 2007).

### Application of the model and testing the refraction identifiability

We have model, in which we assume that it has normal distribution and we consider the impact of refraction  $r$  on measurement of horizontal angles:

$$\begin{pmatrix} \delta\hat{\beta} \\ \delta\hat{\alpha} \\ \hat{r} \end{pmatrix} \sim N_n \left[ \begin{pmatrix} \delta\beta \\ \delta\alpha \\ r \end{pmatrix}; \begin{pmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{pmatrix} \right] \quad (14)$$

and after simplification we can write for the estimate of unknown parameters:

$$\delta\hat{\mathcal{G}} \sim N_n(\delta\mathcal{G}; W) \quad (15)$$

If  $W$  is positively definitive and there exists a matrix  $C_{s,n}$  where  $r(C_{s,n}) = c \leq n$  and matrix  $CWC^T$  is regular, then we can define:

$$(C\delta\hat{\mathcal{G}} - C\delta\mathcal{G})^T [CWC^T]^{-1} (C\delta\hat{\mathcal{G}} - C\delta\mathcal{G}) \sim \chi_s^2(0). \quad (16)$$

Assuming a hypothesis that  $r^* = 0$  ( $r^*$  - vector of real values of refraction), then  $C = (0, 0, I)$ ,  $C\delta\hat{\mathcal{G}} = r$ .

Null hypothesis about refusing the appearance of refraction can be written as follows:

$$\hat{r}^T [(33)]^{-1} \hat{r} \stackrel{H_0}{\sim} \chi_p^2(0) \quad (17)$$

where  $\chi_p^2(0)$  is a random variable with chi-square distribution and  $p$  number degrees of freedom, where  $p$  is a count of considered refraction coeffi-

cients in the control network.

1. Question: How big must be  $r$  to be identifiable by our measurements?

When preparing alternative hypothesis we assume, that  $r \neq 0$  and thus we can write alternative hypothesis as follows:

$$r^T [(33)]^{-1} r = \delta \quad (18)$$

where  $r$  is real value of refraction and  $\delta$  is a parameter of non-centrality  $\chi_p^2$  expression of probability.

The probability of identification  $R(\delta)$  on none-zero vector  $r$  at given  $\delta$  we can calculate according (Kubáčková, Kubáček, 2000) using approximation of random variable  $\chi_p^2(\delta)$  by random variable  $c^2 \chi_f^2(0)$

2. Question: How accurate must be measurement to be able to identify value of particular refraction  $r$ ?

Here we utilize knowledge from the theory of optimal design of experiment, which is described in (Bajtala, 2006), where we take into account the inequality:

$$(33)^{ii} \leq \frac{\delta^*}{r_i^2} \quad (19)$$

where  $\delta^*$  is the wanted value, which is determined by calculation of the null hypothesis for selected threshold region  $R$ .

Since there is a group of refraction angle values  $(r_1, r_2, \dots, r_q)$ , then we can use D-optimal, respectively restricted D-optimal design of experiment. This would define the basic issue for solving the appearance of refraction in the control network.

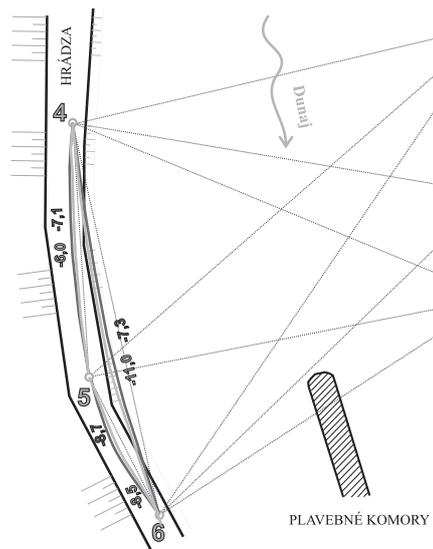


Fig. 6. Graphic interpretation of identifiability of refraction angle  
 Rys. 6. Interpretacja graficzna identyfikowalności kąta załamania

The mentioned equations show several reasons, that emphasize the importance of the best knowledge of the covariance matrix. That is the basis for construction of confidence regions, threshold regions and hypothesis testing. Its imperfect knowledge can lead to incorrect decisions, to incorrect characterization of the accuracy of refraction estimates, what can discredit the work of observer but also cause damage in application and interpretation of applied model. Therefore, it is still necessary to estimate the variance components of applied instruments, which characterized the accuracy of measurement in the control network. This issue is described in more detail in (Bajtala 2006).

### Experimental measurement

Experimental measurements were performed in the area of Gabčíkovo water works, which is situated in the upper part of the Danube middle stream and in the lower part of a supply channel near Gabčíkovo village. The measurement was performed at six points of the control network in the vicinity of locks where is the greatest transverse distance between the points. Three points were located on the left side of the dam crown and three points were located on the right side (fig. 5a). Moreover, we realized measurement on the dam crown and below the dam crown of the supply channel between Vojka nad Dunajom and Kyselica village. In this part of channel operates a ferry. In this case the control network was formed by nine points with two points be-

low the dam crown, four points on the right side and three points on the left side of the dam crown (fig. 5b).

The experimental measurements were performed in different seasons and under different atmospheric conditions. The measurements were not performed in the summer during high temperatures due to large vibrations affecting sights above the dam crown. We carried out four independent measurements in the first stage (E1, E2, E3, E4).

Considering that the primary objective was to assess the impact of the horizontal refraction on the measurements of the horizontal directions, we focused in the first stage of experiments (E1 – E4) to answer the question: how big is the value of horizontal refraction, or if some value of refraction is detected whether it can be identify as refraction or only as random variable caused by the accuracy of instrument or measurement method. The resulting values of horizontal refraction for experiments E1 – E4 are given in tab. 1.

In the second stage of experimental measurements we carried out two additional measurements E5 and E5S. These experiments were performed during one day and during the experiment E5S was at point 6 simulated a shift using a special device. The measurement was realized in May under moderate wind and partly cloudy weather. The temperature ranged between 20–24°C, air pressure between 1005–1007 hPa and humidity 67–70%.

In terms of the accuracy of the resulting estimates obtained by processing of measurement results using the model described above, we first

Tab. 3. Comparison of coordinates before and after simulation: E5 – E5S

Tab. 3. Porównanie współrzędnych przed i po symulacji: E5-E5S

<b>The estimation accuracy of the coordinates and characteristics MODEL WITHOUT refraction - COMPARISON</b>						
Point	E5		E5S		E5S – E5	
	Y X	$\sigma Y$ $\sigma X$	Y X	$\sigma Y$ $\sigma X$	Difference is to be	
	[m]	[mm]	[m]	[mm]	[mm]	[mm]
1	1000,0000	0,0	1000,0000	0,0	0,0	0,0
	1000,0000	0,0	1000,0000	0,0	0,0	0,0
2	1005,9613	0,7	1005,9614	0,8	0,1	0,0
	1329,0728	0,6	1329,0730	0,8	0,2	0,0
3	1000,0000	0,0	1000,0000	0,0	0,0	0,0
	1529,2985	0,6	1529,2994	0,8	0,9	0,0
4	1811,4244	0,6	1811,4248	0,8	0,4	0,0
	1189,5503	1,4	1189,5490	1,7	-1,3	0,0
5	1780,6343	0,9	1780,6349	1,1	0,6	0,0
	1678,2162	1,3	1678,2137	1,7	-2,5	0,0
6	1645,2994	1,2	1645,3026	1,5	3,2	2,0
	1946,3705	1,1	1946,3747	1,5	4,2	6,0

Tab. 4 Comparison of coordinates before and after simulation: E5 – E5S

Tab. 4. Porównanie współrzędnych przed i po symulacji: E5-E5S

<b>The estimation accuracy of the coordinates and characteristics MODEL with refraction - COMPARISON</b>						
Point	E5		E5S		E5S – E5	
	Y X	$\sigma Y$ $\sigma X$	Y X	$\sigma Y$ $\sigma X$	Difference is to be	
	[m]	[mm]	[m]	[mm]	[mm]	[mm]
1	1000,0000	0,0	1000,0000	0,0	0,0	0,0
	1000,0000	0,0	1000,0000	0,0	0,0	0,0
2	1005,9619	0,5	1005,9618	0,5	-0,1	0,0
	1329,0728	0,6	1329,0730	0,7	0,2	0,0
3	1000,0000	0,0	1000,0000	0,0	0,0	0,0
	1529,2984	0,6	1529,2990	0,7	0,6	0,0
4	1811,4247	0,7	1811,4250	0,9	0,3	0,0
	1189,5503	1,4	1189,5503	1,8	0,0	0,0
5	1780,6339	0,8	1780,6336	1,0	-0,3	0,0
	1678,2162	1,4	1678,2148	1,7	-1,4	0,0
6	1645,2997	1,3	1645,3022	1,6	2,5	2,0
	1946,3707	1,2	1946,3760	1,4	5,3	6,0

calculated variance components of applied instruments characterising accuracy the accuracy of the experimental measurements.

In further processing we calculated estimates of refraction angles pertaining to particular horizontal directions. Refraction angles of opposite sights have the same value but different sign. From the estimated covariance matrix of refraction we calculated accuracy characteristics of refraction angles. Subsequently we tested their significance. We calculated individual coefficients of identifiability of refraction  $k_r$ , which were compared with critical value  $k_{r,critical} = 3,84$

calculated for a single measurement  $\chi_1^2(\delta)$  with risk of error 5% (level of significance  $\alpha = 95\%$ ). Thus we obtained significance of the individual refraction angles, which were within the region of identifiability and which were outside the region of identifiability. Graphical representation of identified refraction angles is shown on fig. 6.

For mentioned experiments we estimated coordinates and their accuracy as well and we compared the results processed in model with consideration of horizontal refraction and without consideration of horizontal refraction.

Tab. 5. Estimated refraction angles

Tab. 5. Określone kąty załamania

Directions	Refractions E6, E7		Standard deviation E6, E7		Coefficient of identification $k_{critical} = 3,84$ E6, E7	
	$r$ [°]	$r$ [°]	$\sigma_r$ [°]	$\sigma_r$ [°]	$k_r$	$k_r$
1-2	-2,4	1,6	1,4	1,2	1,0	1,7
1-3	-1,8	1,2	2,1	1,9	0,1	0,4
1-4	-0,3	3,6	1,7	1,5	0,2	5,9
1-5	0,6	3,3	1,7	1,6	0,9	4,4
1-6	-0,5	3,0	2,1	1,9	0,0	2,4
2-3	-0,9	0,2	1,1	1,0	0,0	0,0
2-4	1,6	2,5	1,2	1,1	4,9	5,5
2-5	2,2	3,7	1,3	1,2	5,6	9,5
2-6	-0,5	5,7	1,6	1,5	0,1	14,8
3-4	-2,8	7,2	1,4	1,3	1,5	31,4
3-5	-2,5	5,8	1,3	1,2	1,3	23,0
3-6	-5,6	9,6	1,4	1,3	10,3	54,0
3-7	-1,4	1,2	1,8	1,6	0,0	0,5
3-8	-1,7	1,0	1,4	1,2	0,2	0,7
3-9	-2,7	1,1	1,4	1,3	1,3	0,7
4-5	0,8	-5,2	1,1	1,0	2,9	30,1
4-6	-4,5	3,5	1,1	1,0	10,5	12,9
4-7	-3,0	-1,1	1,6	1,4	1,6	0,6
4-8	-3,1	-1,5	1,2	1,1	2,8	1,8
4-9	-4,6	-1,0	1,3	1,2	7,0	0,7
5-6	-5,4	3,9	1,0	0,9	21,1	20,4
5-7	-3,0	0,2	1,5	1,3	1,8	0,0
5-8	-3,1	0,7	1,3	1,1	2,7	0,3
5-9	-5,1	1,3	1,4	1,3	8,5	1,1
6-7	-1,5	-1,0	1,4	1,3	0,1	0,6
6-8	-0,7	-0,3	1,4	1,3	0,1	0,1
6-9	-6,0	0,2	1,7	1,6	8,4	0,0
7-8	-4,1	-8,8	1,5	1,4	4,2	40,2
7-9	-4,5	-12,6	1,6	1,5	4,6	74,0
8-9	-6,2	1,4	1,5	1,3	12,8	1,1

In the second stage of experimental measurements (E5, E5S), we focused on realization of experiments aimed at acquirement of the refraction angles and assessment of their significance in terms of resulting coordinates of control network points. Therefore, we simulated a shift at point 6. For this purpose we used a special equipment allowing shift with precision 0,05 mm. Simulation during the E5 and E5S experiment was performed directly in the field at point 6 (the farthest point from fixed point 1), 2 mm in the Y axis direction (approximately representing value of standard deviations) and 6 mm in the X axis direction. Measurement with and without simulation was performed within one day.

As in the previous experiments, we estimated variance components of the instruments, refraction angles and their accuracy characteristics together with coefficients of identifiability. By

processing of experiment E5 with model with consideration of horizontal refraction and without consideration of horizontal refraction we obtained resulting coordinates, which were compared. Similarly, we compared coordinates for experiment E5S. We also compared differences in coordinates, calculated with model without consideration of horizontal refraction before and after simulation and with model with consideration of horizontal refraction before and after simulation with real values of shift.

In the third stage of experimental measurements we performed additional two independent measurements E6 and E7 in other location of supply channel (control network formed by nine points). These experiments served for verification or confirmation knowledge obtained from previous experiments. Experiment E6 was performed in November, under cloudy weather with tem-

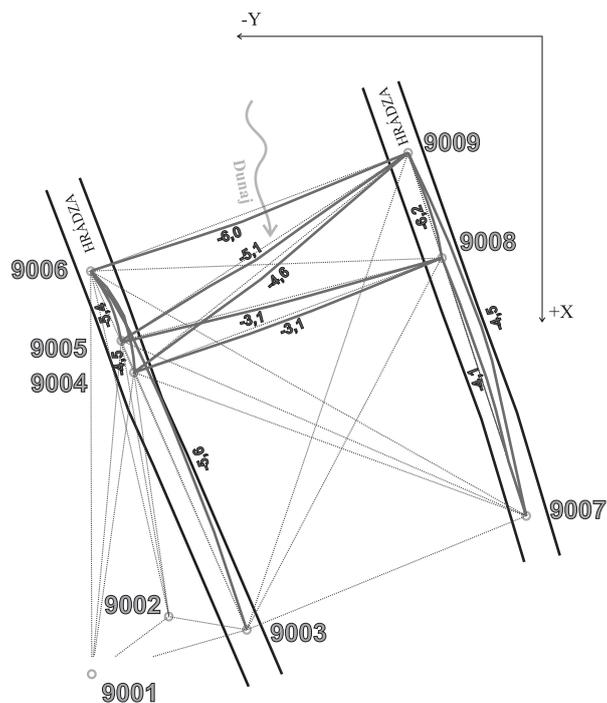


Fig. 7. Graphic interpretation of identifiability of refraction angle: E6  
 Rys. 7. Interpretacja graficzna identyfikowalności kąta załamania: E6

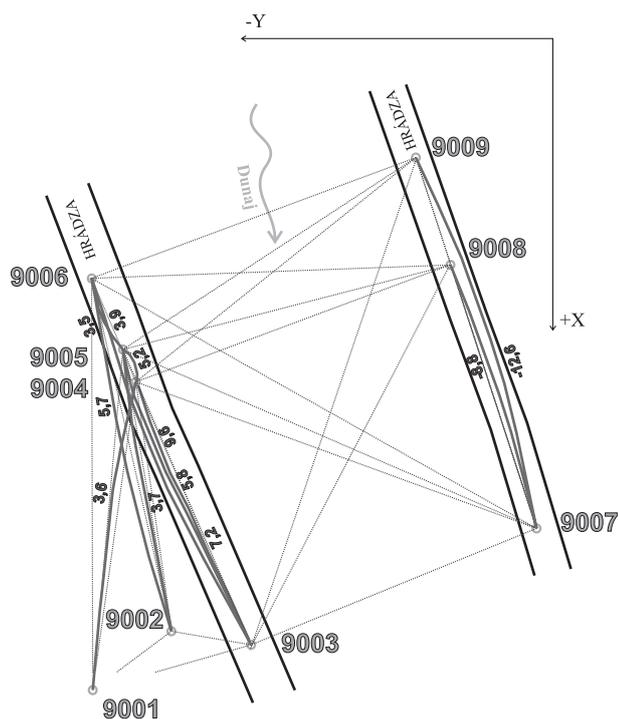


Fig. 8. Graphic interpretation of identifiability of refraction angle: E7  
 Rys. 8. Interpretacja graficzna identyfikowalności kąta załamania: E7

perature from 3 to 6°C, air pressure 1017 hPa and humidity 55–75%. Experiment E7 was performed in the spring in April under sunny weather with temperature from 8 to 18°C, air pressure 996–997 hPa and humidity 45–53%. Estimated refraction angles and their graphical presentation is given in

tab. 5 and fig. 7 and 8. In the third stage of experimental measurements we performed additional two independent measurements E6 and E7 in other location of supply channel (control network formed by nine points). These experiments served for verification or confirmation knowledge ob-

tained from previous experiments. Experiment E6 was performed in November, under cloudy weather with temperature from 3 to 6°C, air pressure 1017 hPa and humidity 55–75%. Experiment E7 was performed in the spring in April under sunny weather with temperature from 8 to 18°C, air pressure 996–997 hPa and humidity 45–53%. Estimated refraction angles and their graphical presentation is given in tab. 5 and fig. 7 and 8.

## Conclusion

Submitted contribution deals with the issue of horizontal refraction in the measurement of local control network and is aimed at confirmation of theoretical knowledge about the occurrence of horizontal refraction in area of water construction. Based on the developed theoretical assumptions, we created a model of refraction, in which questions of occurrence and identifiability of horizontal refraction were solved. Necessary part of this model is also issue related to optimal design and variance components estimation which plays significant role in terms of the accuracy.

Based on the results obtained from experiments E1–E7 we can conclude, that horizontal refraction was identified mainly on the sights passing near the dam crown. This confirms the theoretical assumption of occurrence of horizontal refraction on sights passing above the river bank and valleys. Horizontal refraction was identified on sights above the water level only in experiments performed in spring or winter under cloudy weather with slowly moving regions of water vapour in the locality.

Another important fact resulting from the experiments is that the accuracy of estimated vari-

ance components in model with consideration of horizontal refraction is higher than in the model without consideration of the horizontal refraction. This is caused by the fact that model with horizontal refraction take into account the horizontal refraction and thus to estimation does not enter total value of uncertainty of a particular direction but only is random component. In the model without consideration of the horizontal refraction enter total value of uncertainty and therefore the resulting accuracy of horizontal directions is lower.

Comparing the match of simulated shift and shift calculated as difference in the estimated coordinates before and after simulation shows that model with consideration of horizontal refraction is closer to the real value as the model without consideration of horizontal refraction. Differences from the real shift value were at the edge of standard deviations in the case of model without horizontal refraction, while when considering horizontal refraction the differences were below the standard deviations.

From the practical point of view, realization of further experiments can confirm justification of considering the impact of the horizontal refraction when dealing with challenging tasks in terms of high accuracy requirements and result interpretation, as was confirmed by experiments E6 and E7.

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### Ocena załamania poziomego w sieci geodezyjnej

Proponowany temat dotyczy badania poziomego załamania podczas pomiaru w lokalnych sieciach geodezyjnych. Bazując na opracowanych teoretycznych założeniach zastosowano model załamania oparty na rozwiązywaniu problemów poprzez identyfikację załamań poziomych. Do wdrożenia modelu konieczna jest optymalizacja sieci geodezyjnych oraz zmniejszenie składników wariancji, która pełni bardzo dużą rolę w kwestii precyzji pomiaru. Niezależne podejście do tematu leży w zakresie modelowania załamania poziomego za pomocą podejścia stochastycznego, gdzie wartości załamania wyznacza się opierając się na poziomych danych pomiarowych. W modelu koniecznym jest aby zidentyfikować parametry obliczane na podstawie wszystkich pomiarów oraz pomiędzy dwoma punktami. Na podstawie osiągniętych wyników można stwierdzić, że załamanie poziome zostało stwierdzone głównie dla pomiarów przeprowadzonych blisko budowli piętrzących dla większości przeprowadzonych eksperymentów. Pomiaru zostały potwierdzone na podstawie założeń podanych w części teoretycznej, gdzie efekt załamania poziomego został zmierzony blisko budowli piętrzących, rzeki, wybrzeży oraz dolin.

Słowa klucze: załamanie poziome, podmacierz składników załamania, współczynnik identyfikacji załamania, ocena kąta załamania