The Possibilities of Utilising Postoptimal Analysis for the Decision-Making on the Trends and Concentration of Coal Sales

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Abstract

When developing optimal coal production and sales plans for coal mines, one is often faced with the necessity to modify them, which implies the rationality of such plans. This is achieved through postoptimal analysis, which allows coal mines' production plans, formally optimal, to be modified. The article presents the possibilities of utilising postoptimal analysis developed as part of a method for the rationalisation of production decisions with regard to the management of a coal company. The algorithms resulting from this analysis, accompanied by examples of their practical application, illustrate the possibility of presenting the economic effects of adjustments, if any, quantitatively, which also includes adapting the coal production and sales plans to actual demand, both in terms of quantity and quality. The provided examples of adjustments to the optimal plan concern the "producer-recipient" relationship and the concentration of coal sales.

Keywords: *optimization, post-optimal analysis, Simplex algorithm*

General description of the proposed approach

The developed production-rationalisation approach is a combination of the results of optimising coal production and sales programmes (using the SIMPLEX algorithm) with the algorithmically developed multi-aspect post-optimal analysis. The optimisation model developed and adapted to the conditions of a group of mines (companies) is as follows [1, 2, 3, 4, 5, 6, 7, 8]:

Objective function (quality coefficient):

$$F = \sum_{i=1}^{r_j} \sum_{j=1}^{p} \sum_{k=1}^{m_{ij}} (c_{ijk} - kz_{ijk}) \cdot x_{ijk} - \sum_{j=1}^{p} Ks_j \rightarrow \max \qquad (1)$$

Constraints:

$$\sum_{i=1}^{r_j} \sum_{j=1}^{p} \sum_{k=1}^{m_{ij}} x_{ijk_n} \le Z_k \qquad \text{dla każdego } k, \qquad (2)$$

$$\sum_{i=1}^{r_j} \sum_{n=1}^{m_{ij}} x_{ijk_n} \cdot b_{ijk_n} \le Qs_j \qquad \text{dla każdego } j, \qquad (3)$$

$$\sum_{i=1}^{r_j} \beta_{ij} = 1 \qquad \text{dla każdego } j, \qquad (4)$$

$$x_{ijk} \ge 0 , \qquad (5)$$

where:

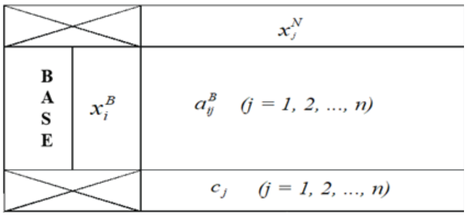
c_{ijk} – price of the *ij*-type of coal accepted by the *k* demand group;
kz_{ijk} – unit variable cost of the *i* type of coal in the conditions of the *j* mine;
Ks_j – total fixed cost of production in the conditions of the *j* mine;
x_{ijk} – net production of the *ij*-type of coal accepted by the k_n demand group;
Z_k – demand of the *k* group of recipients;
Qs_j – total aggregate gross production of the *j* mine;
i – coal type index, *i* = 1, 2, ..., r_j,
j – mine index; *j* = 1, 2, ..., p,
k_n – demand group index; *k* = 1, 2, ..., m_{ij}, where m_{ij} means the size of the k_n set for *ij* type of coal;
b_{ij} – gross/net conversion factor;
β_{ij} – the share of the production of a given type of coal in the total gross production of the mine.

What is important is that in order to accurately reflect the phenomenon of underutilisation of the production capacities typical in market and competition conditions, in each case the criterion function must take into account the division of total costs into fixed and variable costs. Given the interests of any mining company operating in the current market conditions, the most appropriate and viable optimisation is one based on the profit criterion, as it allows the company to refrain from fully meeting the demand unless it is profitable. This can be formally factored in in the optimisation task by placing inequality constraints (2).

The above model leads to a solution in the form of an annual optimal production plan for the company. Although formally optimal (in terms of the linear quality coefficient), the resulting solution does not necessarily have to be the most advantageous from the point of view of the company's interests. At this point, it is necessary to analyse the effects of the desirable optimal-plan adjustments that would make it possible to rationally revise the plan given the prevailing conditions. Adjustments to the optimal plan are made as part of the post-optimal analysis, which constitutes a multi-faceted tool allowing for the fulfilment of the practical conditions mentioned in [3] that are relevant from the decision-maker's point of view. The author confined himself to presenting the algorithm of the adjustment procedure (related to the subject of this publication) along with the numerical example of how the procedure can be used in practice.

The proposed scope of postoptimal analysis

The postoptimal analysis discussed in this publication includes the exploration of how changes to decision variables



Rys. 1. Ogólna postać tablicy SIMPLEX; Źródło: opracowanie własne
Fig. 1. The general form of the SIMPLEX table; Source: Own elaboration

impact on the effect of optimisation, based on the results of the SIMPLEX algorithm. This in practice entails the possibility of accounting for additional important factors, such as the relationships between the producer and the recipient.

The postoptimal analysis therefore allows one to determine which coal production and sales programme will be rational in specific conditions.

The analysis is based on the data obtained from the SIMPLEX algorithm (specifically the SIMPLEX final table) and the values of underlying variables.

The SIMPLEX table offers a complete set of accounting equations and coefficients of goal function sensitivity to changes in the decision variables. The basic form of the SIMPLEX table is shown in Fig. 1.

The key to the figure is as follows:
a_{ij}^B – constraint coefficients forming the *A* matrix;
x^B, x^N – vectors of basic and nonbasic decision variables, respectively;
c – vector of objective-function coefficients (of shadow prices).

The formal starting point for the post-optimal analysis is, therefore, the optimal solution, which – in relation to the basic and nonbasic variables and the quality coefficient – is represented by the following equations [4, 5, 6, 7, 9]:

$$x^B = [A^B]^{-1} \cdot B - [A^B]^{-1} \cdot A^N \cdot x^N \quad (6)$$

$$J = c^{BT} \cdot [A^B]^{-1} \cdot B - [[c^{BT} \cdot [A^B]^{-1} \cdot A^N]^T - c^N]^T \cdot x^N \quad (7)$$

where:
A^B, A^N – submatrixes of the *A* matrix (*A* – matrix of the constraint coefficients);
B – vector of the right-hand sides of the equation;
c^B, c^N – subvectors of objective-function coefficients;
J – objective function (quality coefficient).

The post-optimal analysis will directly use the formulas obtained after substitutions and reductions [1, 3, 4, 5, 6]:

$$x^B = x^{BO} = x^{BO} - A^O \cdot x^N \quad (8)$$

and

$$J = J^O - c^{OT} \cdot x^N \quad (9)$$

where:
x^{BO} – vector of the optimal values of basic variables;
c^O – shadow prices of nonbasic variables, ≥ 0 for maximisation of the quality coefficient and negative for minimisation;
A^O – matrix of optimal-solution coefficients;
J^O – optimal value of the quality coefficient.

The post-optimal analysis can be used to change selected decision variables while maintaining the feasibility of the solution, i.e. maintaining the positive values of all variables and taking into account their mutual relations expressed with the formula (8). As indicated by the relationship (9), the shadow prices can be used to estimate the economic effects of departing from the optimal solution as a result of an increase in nonbasic variables [1, 3, 5]. What is also important is that the adjustments of production plans can be made without having to solve the problem (start the optimisation procedure) again from the beginning, substantially reducing the calculation time.

The algorithm for incorporating producer-recipient relationships

If some of the non-underlying variables have zero shadow prices, they can be modified without any losses to the quality indicator [equation (9)]. Such ambiguities in the optimisation solution are often encountered while planning coal production. This provides the decision-maker with a certain degree of freedom when it comes to establishing the final structure of coal production and sales (e.g. by taking into account the existing producer-recipient relationships and concentration of sales directions). Should the adjustment generate losses, postoptimal analysis will make it possible to assess their validity by comparing them with the benefits resulting from the modification of the plan.

An indisputable benefit for a coal mine which is linked to a specific recipient lies in the fact that the mine acquires a regular customer for its product (e.g. through long-term contracts) and can negotiate favourable coal prices (e.g. the need to adjust the quality of production to the recipient's requirements). The strategic recipients of the mining industry are power plants and CHP plants. Such a solution is beneficial for the recipient also due to there being fewer coal acquisition "channels", which lowers the related costs of transport. At the same time, a need may arise to restrict sales to other customers or even forgo some of them (this is true in the case of modifying the obtained solution for an optimal coal production and sales plan, while retaining its optimality). In such a case, regaining "lost" customers may prove difficult or even impossible. For this reason, when cooperating with only several customers, the coal mine should reconsider whether this strategy is profitable. An undoubtedly adverse effect of this decision occurs if the recipient associated with the mine is forced to reduce its demand or goes into liquidation. The situations described above are obviously extreme cases that were only mentioned to make the reader aware of the problem.

From a computational point of view, this strategy boils down to finding, according to the equation:

Tab. 1. Optymalny plan produkcji po korekcie „powiązanie producent–odbiorca"; Źródło: opracowanie własne

Tab. 1. An optimal production plan following the adjustment of the "producer-recipient relationship"; Source: Own elaboration

Company „Alpha"				
Max. Extraction: 15,949,350 ton		Profit: 316,267,643 zł		
Sold: 11,423,865 ton		Company reserves: 1,854,588 ton		
Mine „A"				
Max. Extraction: 1,454,750 ton		Profit: 4,806,243 zł		
Sold: 597,902 ton		Mine reserves: 0 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase − decrease [ton]
Dust kettles	fine coal I	264,765	264,765	0
Dust kettles	fine coal II	317,135	317,135	0
Grates 4	slurry	16,002	16,002	0
Dumping coal	cobble	160,023	160,023	0
Dumping coal	nut coal	21,821	21,821	0
Dumping coal	fine coal IIA	675,004	675,004	0
Mine „B"				
Max. Extraction: 793,500 ton		Loss: −4,352,243 zł		
Sold: 121,880 ton		Mine reserves: 400,338 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase − decrease [ton]
Grates 3	fine coal II	121,880	113,486	8,394
Dumping coal	coaking coal	271,282	252,598	18,685
Mine „C"				
Max. Extraction: 1,110,900 ton		Profit: 4,8,896,429 zł		
Sold: 989,348 ton		Mine reserves: 121,552 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase − decrease [ton]
Export 5	coaking coal	123,542	123,542	0
Coking plants 3	coaking coal	865,806	865,806	0
Mine „D"				
Max. Extraction: 3,174,000 ton		Profit: 53,348,590 zł		
Sold: 1,653,633 ton		Mine reserves: 1,501,900 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase − decrease [ton]
Export 2	coaking coal	302,606	292,713	9,893
Export 3	coaking coal	220,455	220,455	0
Indv. consumers 2	cobble	36,934	36,934	0
Indv. consumers 3	fine coal IIA	637,566	637,566	0
Grates 3	fine coal II	0	8,394	-8,394
Coking plants 1	coaking coal	408,682	418,575	-9,893
Chamber grates 1	fine coal IIA	47,390	47,390	0
Dumping coal	fine coal I	18,467	18,467	0
Mine „E"				
Max. Extraction: 2,988,850 ton		Profit: 73,490,006 zł		
Sold: 2,934,441 ton		Mine reserves: 9,893 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase − decrease [ton]
Export 1	coaking coal	22,984	22,984	0
Export 2	coaking coal	0	9,893	-9,893
Export 8	nut coal	38,855	38,855	0
Indv. consumers 2	cobble	215,197	215,197	0
Dust kettles	fine coal I	206,231	206,231	0
Dust kettles	fine coal IIA	1,545,235	1,545,235	0

Dust kettles	fine coal II	863,778	863,778	0
Grates 4	slurry	13,752	13,752	0
Chamber grates 2	slurry	28,409	36,803	-8,394
Dumping coal	slurry	44,516	36,122	8,394
Mine „F"				
Max. Extraction: 3,385,600 ton			Profit: 105,742,684 zł	
Sold: 3,069,123 ton			Mine reserves: 0 ton	
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 7	cobble	225,012	225,012	0
Coking plants 2	coaking coal	68,243	78,136	-9,893
Export 8	nut coal II	46,885	46,885	0
Indv. consumers 2	cobble	18,507	18,507	0
Coking plants 1	coaking coal	241,457	231,563	9,894
Dust kettles	fine coal I	243,520	243,520	0
Dust kettles	fine coal IIA	23,675	23,675	0
Dust kettles	fine coal II	2,201,825	2,201,825	0
Dumping coal	coaking coal	228,073	228,073	0
Dumping coal	nut coal	13,995	13,995	0
Dumping coal	slurry	74,409	74,409	0
Mine „G"				
Max. Extraction: 3,041,750 ton			Profit: 34,335,366 zł	
Sold: 1,915,250ton			Mine reserves: 520 ton	
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Indv. consumers 2	cobble	53,039	53,039	0
Export 9	fine coal IIA	933,003	932,482	521
Export 9	fine coal II	206,112	206,632	-520
Indv. consumers 3	fine coal IIA	677,516	677,516	0
Dust kettles	fine coal IIA	0	521	-521
Dust kettles	fine coal II	12,155	12,155	0
Chamber grates 2	fine coal II	33,426	33,426	0
Grates 4	slurry	53,315	53,315	0
Dumping coal	nut coal	15,194	15,194	0
Dumping coal	coaking coal	1,057,470	1,057,470	0

$$x_i^B + a_{ik}^o \cdot x_k^N \geq 0 \quad (10)$$

such a nonbase variable that links the mines to a specific recipient accepting the type of coal offered.

The balance relation between the nonbasic variable and basic variables based on the coefficients of a selected SIMPLEX tableau column is as follows:

$$x_i^B = \bar{x}_i^B + a_{ik}^O \cdot x_k^N \quad (11)$$

where: $\bar{x}_i^B$– a new adjusted value of the basic variable.

After adjusting the defined variable value x_j^N, the new basic variables will take the following form:

$$\bar{x}_i^B = x_i^B - a_{ik}^O \cdot x_k^N \quad (12)$$

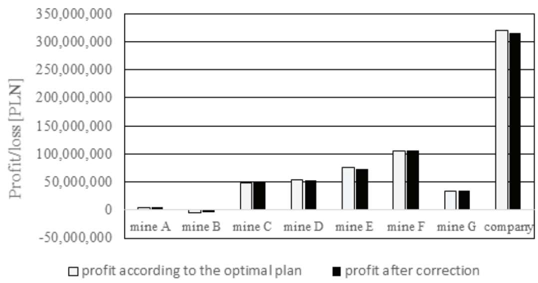
The calculation procedure for the proposed strategy is as follows [3]:
1. From the system of equations (12), the one is chosen for which the quotient:

$$\frac{x_i^B}{a_{ik}^O} > 0 \quad (13)$$

is the smallest and positive. It is the maximum value by which it is possible to increase the nonbasic variable without exceeding the constraints of the model.

2. If the change is satisfactory to the decision maker, the required adjustment to the i basic variable is made by increasing the k nonbasic variable by the value $\frac{x_i^B}{a_{ik}^O}$. This yields a minimum decrease in the value of the quality coefficient. In the case of thus determined value of the nonbasic variable, the remaining values of the basic variables are calculated according to the formula (12), and the calculation procedure is completed.

Following the steps in points 1 and 2 of the strategy in question, one can find the maximum value of the non-underlying variable (production volume), which does not affect the restrictions of the task and causes a minimal change to the quality indicator. It is by this volume, or the volume assumed by the decision-maker, that the underlying vari-



Rys. 2. Zysk/strata spółki i kopalń według planu optymalnego i po korekcie powiązania producent–odbiorca; Źródło: opracowanie własne
Fig. 2. Profit/loss of the company and coal mines according to the optimal plan and after adjusting the producer-recipient relationship; Source: Own elaboration

able (sales) which captures the link between the coal mine and the recipient, is increased, and the other underlying variables are adjusted. The condition to be met in order for a decision-maker to create an exclusive link between a specific recipient and a specific mine is that the mine is able to satisfy that decision-maker's needs in both quantitative and qualitative terms. Once this condition is met, the calculation procedure presented above will essentially consist of a search through all non-underlying variables linked to a given coal mine and mines which sell coal to the same recipient, and the elimination of their sales. Any resulting losses must be then set by the decision-maker against benefits brought about by the said strategy.

Assessing the effects of the assumed adjustment of the producer-recipient relationship

On analysing the optimal production and sales plan of company *Alfa* (Table 1, column 4), one will notice small sales figures in the following mines:

- coal mine "D" – 8,394 tonnes for the recipient "Grates 3";
- coal mine "E" – 9,893 tonnes for the recipient "Export 2";
- coal mine "F" – 521 tonnes for the recipient "Dust kettles".

Supplying such small amounts of coal to the recipient is not profitable for the sole reason of transport costs. For example, coal mine "B" could increase its production by 8,394 tonnes, because the "Grates 3" group is also a recipient of its coal.

The minimum optimal sales flow was assumed to amount to 12,000 tonnes. For this flow volume, the company's optimal production and sales plan was adjusted in accordance with the above-presented algorithm. The results of the adjustment are shown in Table 1. Figure 2 shows profit/loss trends in the respective coal mines and the company.

The adjustment resulted in the following changes when compared to the optimal plan [3]:

1) In coal mine "B" – increase in sales to the recipient "Grates 3" by 8,394 tonnes, resulting in the removal of this recipient from the sales plan of coal mine "D". In consequence, the sales volume grew by 7.4%, reducing the loss of coal mine "B" by 10%; the amount of unused reserves dropped by 6.3%.

2) In coal mine "D", following the removal of the recipient "Grates 3", the sales volume recorded a 0.5% decrease, causing a profit reduction of 2.18%. The coal mine's reserves increased by the amount of coal sold to the recipient removed from the plan.

3) The production plans of coal mines "A" and "C" remained unchanged.

4) As regards coal mine "E", its sales dropped by 0.62%, resulting in profit lower by 3.48%.

5) The recipient of coal was replaced in the production plans of coal mine "F", which resulted in a slight (by PLN 466) increase in profit.

6) In coal mine "G", the low volume of sales (521 tonnes) for the recipient "Dust kettles" was cancelled in favour of the recipient 'Export 9'. This is also the amount by which the sales of fine coal II assortment dropped for this recipient.

7) The company's profit resulting from this adjustment dropped by 1.07%; sales dropped by 0.7%; production reserves recorded a 0.4% decrease.

Summary

1. The proposed method allows the analysis and evaluation of additional practical aspects deemed relevant, which change over time and which were not included in the general model of optimisation.

2. The presented examples of practical uses of the method illustrate the possibility of depicting the economic effects of adjustments in quantitative terms – this includes adapting coal production and sales plans to actual changes in both the level and structure of demand.

3. The coal production and sales programmes, which meet the adopted optimisation criterion in adjusted (through post-optimal analysis) conditions, are deemed rational in the decision-making scenario in question.

4. The presented method facilitates the adaptation of production decisions to relevant, both internal and external, additional conditions, as well as their changes, while fully accounting for the effects of alternative decisions under consideration.

The paper presents results of research conducted in AGH University of Science and Technology no. 16.16.100.215

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Możliwości wykorzystania analizy postoptymalnej do podejmowania decyzji o kierunkach i koncentracji zbytu węgla

Przy opracowywaniu optymalnych programów produkcji i sprzedaży węgla dla kopalń występuje niejednokrotnie konieczność ich modyfikacji, co implikuje racjonalność planów produkcji i sprzedaży węgla. Realizuje się to dzięki analizie postoptymalnej, pozwalającej na modyfikację formalnie optymalnych planów produkcyjnych kopalń. W artykule zaprezentowano możliwości analizy postoptymalnej opracowanej w ramach metody racjonalizacji decyzji produkcyjnych dla potrzeb zarządzania spółką węglową. Opracowane w ramach tej analizy algorytmy poparte przykładami praktycznego ich wykorzystania ilustrują możliwości ilościowego ujmowania skutków ekonomicznych ewentualnych korekt, w tym dostosowania planów produkcji i sprzedaży węgla do realnych zmian zapotrzebowania, zarówno w sensie ilościowym jak i jakościowym. Podane przykłady korekt planu optymalnego dotyczą powiązania producent-odbiorca oraz koncentracji zbytu węgla.

Słowa kluczowe: *optymalizacja, analiza postoptymalna, algorytm Simpleks*