Factor Analysis and Mathematical Modeling in Determining the Quality of Coal

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Abstract

The separation of coal material of three types of coals originating from three various Polish hard coal mines (types 31, 34.2 and 35, according to Polish nomenclature, which were steam coal, semi-coking coal and coking coal) into particle size fractions and then into particle density fractions was done and then the following parameters were measured for each particle size-density fraction: combustion heat, ash contents, sulfur contents, volatile parts contents, analytic moisture. In this way a 7-dimensional vector of data was created. Using methods of factor analysis the important features of coal were selected, which decide about their membership to individual types. To evaluate the appropriateness of the applied method the Bartlett's sphericity test as well coefficient of Kaiser-Mayer-Olkin (KMO) were used. To select important factors the Kaiser criterion and Cattell's scree test were used. The obtained results were compared with the results obtained in previous works by means of observation tunnels method. The results showed which particular features are crucial to define the type of coal what is also important to select appropriate method of its enrichment. Furthermore, the construction of a mathematical model presenting the relations between these properties and particle size and density is presented. Because of the fact that particles of certain size or density may occur in neighboring fractions three sorts of relations were examined basing on regression analysis.The analysis was conducted for all three coal types. Because of the fact that the models contain various amounts of independent variables R^2 coefficient, mean squared error (MSE) and Mallow's statistics Cp were applied to evaluate and compare obtained results.

Keywords: *coal, multidimensional statistical analysis, factor analysis, quality of coal, particle size, particle density*

1. Introduction

Mineral raw materials which are beneficiated in purpose of their using characterize with many factors describing their features. In case of coal, these features are among others ash contents, sulfur contents, combustion heat, volatile parts contents or analytic moisture. The features mentioned above decide about coal quality also in economical aspect. Because of that the preciseness of determining values of these features is very important.

The most often researched properties of the coal are combustion heat, ash contents, sulphur contents, volatile parts contents and moisture. These features are very often highly correlated but also can occur independently . The selection of the necessary factors which influence on individual properties is the goal of the paper. To this purpose three types of coal (according to Polish nomenclature – coal types 31 (steam coal), 34.2 (gas-coking coal) and 35 (orto-coking coal)) were selected to the investigation which were divided into particle size and density fractions. The classification of coals is presented in Table 1.

The whole group of considered factors were measured for each size-density fraction [14].

The following variables were considered (X_i = 1, 2, …, 5).

X_1 – combustion heat [cal];
X_2 – ash contents [%];
X_3 – sulfur contents [%];
X_4 – volatile parts contents [Va];
X_5 – moisture [Wa].

Knowledge about these features can serve also to evaluate beneficiation process (Brożek, 1984; Dobosz, 2001; Foszcz et al., 2016; Głowiak 2019a; b; Niedoba, 2013a; Stanisz, 2007; Stępiński, 1964; Tumidajski and Saramak, 2009). The ash contents, sulfur contents and volatile parts contents were investigated dependably on particle size and particle density also by means of kriging method (Niedoba, 2013a). The application of non-conventional statistical methods can be very beneficial in getting precise information (Foszcz et al., 2018; Jamróz, 2009; 2014a; b; c; Jamróz et al., 2016; 2017; Jamróz and Niedoba, 2014; 2015a; b; Niedoba, 2009; 2011; 2013a; b; 2014; 2015; Niedoba and Surowiak, 2012; Niedoba et al., 2018; Öney, 2019; Pięta et al., 2018; Surowiak 2007, 2014). The presented work is an attempt of constructing new mathematical model describing relation between ash contents and particle size and density.

2. Materials and methods

The considered types of coal originated from three various Polish coal mines and all of them were initially screened on a set of sieves of the following sizes: -1.00, -3.15, -6.30, -8.00, -10.00, -12,50, -14.00, -16.00 and -20.00 mm. Then, the size fractions were additionally separated into density fractions by separation in dense media using zinc chloride aqueous solution of various densities (1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 g/cm³). The fractions were used as a basis for further consideration and additional coal features were determined by means of chemical analysis. In purpose of appropriate identification

Tab. 1. Classification of coal types according to Polish nomenclature (Sobolewski et al., 2016)
Tab. 1. Klasyfikacja węgli według polskiej nomenklatury (Sobolewski et al., 2016)

Coal type	Coal number	Volatile parts contents [%]
Steam coal	31	Above 28
Gas-steam coal	32	Above 28
Gas coal	33	Above 28
Gas-coking coal	34	Above 28
Orto-coking coal	35	20-31
Meta-coking coal	36	14-28
Semi-coking coal	37	14-28
Thin coal	38	14-28
Anthracite coal	41	10-14
Anthracite	42	3-10
Meta-anthracite	43	Till 3

Tab. 2. Data for size fraction 14.00-12.50 mm – coal, type 31
Tab. 2. Dane dla klasy ziarnowej 14.00-12.50 – węgiel, typ 31

Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
<1.3	308.6	7048	6.41	0.72	34.32	3.23
1.3-1.4	292.5	5859	19.61	0.7	29.22	3.36
1.4-1.5	36.1	2948	16.55	0.76	28.92	3.87
1.5-1.6	10.7	5117	26.10	1.55	31.08	3.40
1.6-1.7	25.6	4467	35.78	2.28	26.71	2.40
1.7-1.8	139	3920	37.20	1.23	29.24	2.19
1.8-1.9	12.7	3078	48.20	1.13	24.05	2.23
>1.9	601.2	457	86.53	0.40	9.30	0.91

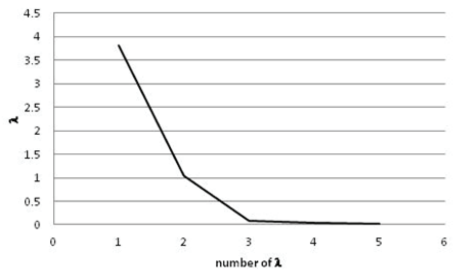

Fig. 1. Cattell's scree plot for coal, type 31, particle size fraction (10-12.5)
Rys. 1. Wykres osypiska Cattella dla węgla, typ 31, klasa ziarnowa (10-12.5)

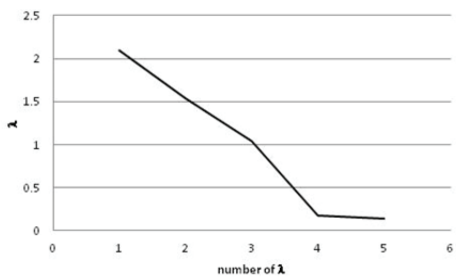

Fig. 2. Cattell's scree plot for coal, type 34.2, particle density fraction (1.6-1.7)
Rys. 2. Wykres osypiska Cattella dla węgla, typ 34.2, frakcja gęstościowa (1.6-1.7)

of coal type many parameters are being measured which describe coal quality. For each density-size fraction such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents and analytical moisture were determined, making up, together with the mass of these fractions, seven various features for each coal.

The example of obtained data is presented in Table 2.

The measurements of X_i were performed for each size-density fraction. Because of the fact that the individual features were measured in various units their standardization was done.

In purpose of selecting significant factors influencing on individual variables, the factor analysis method was applied. To evaluate adequacy of applying factor analysis to this problem two criteria were used: Bartlett's test and Kaiser-Mayer-Olkin coefficient (KMO) (Comrey, 1973; Dobosz, 2001; Kline, 1994;Lawley and Maxwell, 1971; Tumidajski and Saramak, 2009).

The reduction of variables is done through the Cattell's scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [Stanisz, 2007].

Tab. 3. Influence of factors on properties of coal, type 31 by particle size fractions
Tab. 3. Wpływ czynników na właściwości węgla, typ 31 według klas ziarnowych

Feature	**0.5-1**	**1-3.15**	**3.15-6.3**	**6.3-8**	**8-10**	**10-12.5**	**12.5-14**	**14-16**	**16-20**
Combustion heat									
Z_1	97.49	95.39	97.57	97.63	94.13	96.29	81.52	95.41	99.89
Z_2							11.42		
Ash contents									
Z_1	86.47	95.55	90.68	99.34	95.86	96.58	97.33	97.65	99.28
Sulfur contents									
Z_1	95.74	68.65	94.59	7.99	5.00	0.04	2.31	22.45	2.06
Z_2		30.83		91.96	94.63	99.26	90.13	77.28	97.19
Volatile parts contents									
Z_1	94.71	95.60	84.65	88.34	93.41	95.90	96.09	88.56	86.71
Z_2			12.11						
Moisture									
Z_1	93.25	95.21	75.25	95.19	95.90	92.34	79.90	97.35	94.22
Z_2			18.16				7.00		

Tab. 4. Influence of factors on properties of coal, type 34.2 by particle size fractions
Tab. 4. Wpływ czynników na właściwości węgla, typ 34.2 według klas ziarnowych

Feature	**0.5-1**	**1-3.15**	**3.15-6.3**	**6.3-8**	**8-10**	**10-12.5**	**12.5-14**	**14-16**	**16-20**
Combustion heat									
Z_1	97.89	97.73	88.58	65.37	94.14	86.65	82.95	95.90	91.83
Z_2				6.96					
Z_3				27.60					
Ash contents									
Z_1	97.37	96.35	99.52	93.33	94.53	91.83	87.25	91.83	85.34
Sulfur contents									
Z_1	97.51	62.60	1.84	9.92	31.49	54.43	20.79	6.25	52.26
Z_2		16.74	95.29	88.37	65.15	11.59	72.53	91.43	44.28
Z_3		5.85				31.21			
Volatile parts contents									
Z_1	61.29	86.22	92.58	93.91	90.68	77.51	68.14	84.89	82.95
Z_2	25.35					11.59	26.65	0.09	14.36
Z_3								7.59	
Moisture									
Z_1	22.21	49.29	30.33	89.20	70.39	24.35	38.26	64.02	66.19
Z_2	73.23	32.06	3.01		16.54	69.87	49.75	8.34	29.99
Z_3		4.23	66.48					21.64	

Tab. 5. Influence of factors on properties of coal, type 35 by particle size fractions
Tab. 5. Wpływ czynników na właściwości węgla, typ 35 według klas ziarnowych

Feature	**0.5-1**	**1-3.15**	**3.15-6.3**	**6.3-8**	**8-10**	**10-12.5**	**12.5-14**	**14-16**	**16-20**
Combustion heat									
Z_1	92.23	98.46	96.29	92.87	90.70	97.12	97.49	92.14	84.89
Z_2									13.80
Ash contents									
Z_1	92.23	98.01	76.47	95.29	91.75	96.80	96.68	93.89	95.08
Z_2			12.64						
Sulfur contents									
Z_1	12.18	4.59	6.48	1.16	24.70	2.23	0.02	4.32	26.79
Z_2	70.66	77.03	34.07	97.02	52.67	69.80	67.99	75.00	62.85
Z_3	15.35	18.35	48.24		21.51	27.23	31.84	16.42	
Volatile parts contents									
Z_1	78.35	87.90	99.64	92.14	76.51	92.23	84.54	87.60	87.19
Z_2	6.88				13.64		8.93		
Moisture									
Z_1	15.14	36.10	54.49	40.60	44.32	0.04	2.74	1.15	15.94
Z_2	68.49	29.89	31.16	13.81	46.37	89.85	63.66	83.50	72.35
Z_3	14.06	26.68		43.74			33.50	15.26	

3. Results

3.1. Factor analysis

Applying Bartlett's test it occurred that for all researched cases the value of the test was significantly higher than the critical values on significance level being equal to α = 0.0005. The lowest value of the test *U* was obtained for coal, type 35 in particle density fraction (1.9–2.0) and was equal to 84.74, while the critical value on this level is equal to 31.42. It can

Tab. 6. Influence of factors on properties of coal, type 31 by particle density fractions
Tab. 6. Wpływ czynników na właściwości węgla, typ 31 według frakcji gęstościowych

Feature	**<1.3**	**1.3-1.4**	**1.4-1.5**	**1.5-1.6**	**1.6-1.7**	**1.7-1.8**	**1.8-1.9**	**1.9-2.0**
Combustion heat								
Z_1	86.99	87.47	35.10	84.97	59.66	28.46	87.94	75.69
Z_2			60.40	8.15	20.53	63.98		27.82
Z_3					9.15			
Ash contents								
Z_1	92.42	94.03	82.88	70.94	83.86	82.04	75.15	51.62
Z_2			1.33	25.38	11.19	0.02	9.01	36.33
Z_3			3.12			17.05	0.01	
Z_4							15.30	
Sulfur contents								
Z_1	7.82	17.61	35.58	64.03	36.52	18.13	1.60	40.24
Z_2	56.73	80.64	48.87	4.86	36.97	67.04	87.51	54.30
Z_3	34.85		14.49	17.92	23.87			
Volatile parts contents								
Z_1	89.88	87.01	73.80	68.22	21.16	71.84	18.36	74.33
Z_2			0.03	1.26	43.08	13.03	42.04	16.73
Z_3			0.06	6.83	35.45	1.06	38.69	
Z_4			24.75	23.66				
Moisture								
Z_1	1.87	79.85	63.42	46.36	93.10	60.04	39.66	66.11
Z_2	67.24	0.09	5.97	37.93		5.84	34.95	0.07
Z_3	30.74	18.61	27.06	15.37		31.75	0.06	37.06
Z_4							24.35	

Tab. 7. Influence of factors on properties of coal, type 34.2 by particle density fractions
Tab. 7. Wpływ czynników na właściwości węgla, typ 34.2 według frakcji gęstościowych

Feature	**<1.3**	**1.3-1.4**	**1.4-1.5**	**1.5-1.6**	**1.6-1.7**	**1.7-1.8**	**1.8-1.9**	**1.9-2.0**
Combustion heat								
Z_1	68.90	21.37	75.81	52.56	73.37	99.60	91.83	83.37
Z_2	13.54	70.82	1.03	28.72	17.60			7.68
Z_3	14.35		20.48	13.13				
Ash contents								
Z_1	80.94	83.39	89.18	73.41	34.26	13.14	87.25	6.51
Z_2	6.02	10.66		3.69	43.21	23.72		86.19
Z_3	18.01	4.79		9.90	14.92	46.22		
Z_4						5.34		
Sulfur contents								
Z_1	18.36	83.26	63.64	53.96	30.73	51.60	37.05	95.39
Z_2	64.78	11.15	8.70	0.06	56.11	48.00	52.91	
Z_3	16.30		23.41	31.76				
Volatile parts contents								
Z_1	82.88	71.58	56.23	11.26	52.25	1.46	75.65	87.32
Z_2	15.10	0.06	0.01	81.28	34.98	72.72	19.64	
Z_3		26.70	34.32			14.75		
Moisture								
Z_1	49.75	47.22	2.97	43.19	19.22	43.02	40.90	64.67
Z_2	25.45	42.22	86.52	8.72	2.11	32.11	48.26	24.86
Z_3	20.63			39.77	76.65	4.38		
Z_4						20.13		

be said then that zero hypothesis (that correlation matrix is a unit matrix) should be rejected for all particle size and density fractions.

Furthermore, it can be noticed that in almost all cases the value of KMO coefficient was higher than 0.5. Only for density fraction lower than 1.3 g/cm³ for coal, type 34.2 and density fraction (1.6-1.7) for coal, type 35 it occurred to be slightly lower than 0.5. That means that the results of Bartlett's test and the values of KMO coefficient gave strong basis to apply factor analysis.

In the work, the reduction of variables is done through the Cattell's scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [22].

The correlation matrix of the factor Z_j with variable X_i is obtained by creation of matrix Z, which elements are numbers

$$z_{ij} = \sqrt{\lambda_j a_{ji}}, \qquad i, j = 1, 2, \ldots, 5. \tag{1}$$

Tab. 8. Influence of factors on properties of coal, type 35 by particle density fractions
Tab. 8. Wpływ czynników na właściwości węgla, typ 35 według frakcji gęstościowych

Feature	<1.3	1.3-1.4	1.4-1.5	1.5-1.6	1.6-1.7	1.7-1.8	1.8-1.9	1.9-2.0
Combustion heat								
Z_1	36.22	94.80	58.46	93.14	55.65	88.39	59.42	99.70
Z_2	47.32		0.17		43.09		18.46	
Z_3	13.34		37.14				8.12	
Ash contents								
Z_1	36.62	97.91	15.03	94.01	71.14	76.54	77.59	59.87
Z_2	47.32		56.07		23.93	9.48	5.65	32.02
Z_3	13.34		27.41				3.76	
Sulfur contents								
Z_1	22.05	40.51	16.48	9.04	21.90	75.06	27.98	57.54
Z_2	70.30	54.61	37.22	78.17	16.54	10.15	53.01	26.44
Z_3			17.61	7.79	54.39			0.02
Z_4			25.56					
Volatile parts contents								
Z_1	94.78	55.74	4.00	49.97	28.64	39.06	31.14	26.50
Z_2		31.34	42.35	19.51	44.55	35.14	10.68	44.03
Z_3			20.81	15.70	11.12	13.42	51.94	12.22
Z_4			31.70		4.93			9.95
Moisture								
Z_1	70.12	23.27	86.19	7.81	27.96	5.03	52.51	6.35
Z_2	14.68	1.38		33.79	44.98	68.92	18.25	0.21
Z_3	11.48	72.67		48.87	20.76	25.99	1.18	89.98
Z_4							17.51	

Tab. 9. Ash contents by separation in accordance to particle size – coal, type 31
Tab. 9. Zawartość popiołu według rozdziału na klasy ziarnowe – węgiel, typ 31

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
0.750	12.480	0.000	13.550	12.790	11.180	10.530	10.170
2.075	13.550	12.480	15.740	16.110	17.840	14.080	15.020
4.725	15.740	13.550	32.000	22.740	21.950	23.150	22.750
7.150	32.000	15.740	41.940	28.820	26.190	30.640	29.360
9.000	41.940	32.000	42.240	33.460	35.040	35.140	35.560
11.250	42.240	41.940	46.150	39.090	42.020	41.200	42.100
13.250	46.150	42.240	47.480	44.100	44.930	46.240	46.290
15.000	47.480	46.150	50.010	48.480	48.870	50.860	50.720
18.000	50.010	47.480	0.000	56.000	53.580	49.800	49.610

where: λ_i–i^{th} eigenvalue of correlation matrix; a_{ji} – elements of matrix A which fulfills the condition $A^T=R$, where R is correlation matrix of variables X_j.

The square of number z_{ij} is the percentage of variance changeability explained by the factor Z_j. For example, considering coal, type 31 from the particle size fraction (10-12.5) it is obtained that matrix Z is in form

$$Z=\begin{bmatrix} -0.9813 & 0.1331 & -0.0962 & 0.0676 & 0.0747 \\ 0.9828 & -0.1017 & 0.1145 & -0.0767 & 0.0700 \\ -0.0667 & -0.9963 & -0.0484 & 0.0246 & 0.0033 \\ -0.9793 & -0.0651 & -0.0297 & 0.1893 & -0.0019 \\ -0.9620 & -0.1035 & 0.2487 & 0.0063 & -0.0029 \end{bmatrix} \quad (2)$$

The eigenvalues of the correlation matrix are in this case numbers λ_1=3.8177; λ_2=1.0355; λ_3=0.0875; λ_4=0.0488; λ_5=0.0105.

The plot of scree is presented on Figure 1.

On the basis of the presented Cattell's scree plot only these factors remain which are located to the left from the point in which a mild decline of eigenvalues is observed. In this case these are factors Z_1 and Z_2.

The group of factors (Z_1, Z_2) explain 98.07% of changeability of combustion heat, 97.12% of changeability of ash contents, 99.71% of changeability of sulfur contents, 96.33% of changeability of volatile parts contents and 93.62% of changeability of moisture.

It is obtained then that factor Z_1 is responsible for variables {X_1, X_2, X_4, X_5} and factor Z_2 for variable X_3.

Let consider the particle density fraction (1.6-1.7) of coal, type 34.2.

The matrix Z is in form

$$Z=\begin{bmatrix} 0.8566 & -0.4196 & -0.1506 & -0.1824 & 0.3643 \\ -0.5854 & -0.6574 & 0.3863 & 0.1622 & 0.1400 \\ 0.5544 & -0.7491 & -0.2733 & 0.1882 & -0.2073 \\ -0.7229 & -0.5915 & -0.1757 & -0.2615 & -0.0447 \\ -0.4385 & 0.1454 & -0.8755 & 0.0956 & 0.0961 \end{bmatrix} \quad (3)$$

The eigenvalues of correlation matrix in this case are numbers λ_1=2.0993; λ_2=1.5404; λ_3=1.0443; λ_4=0.1727; λ_5=0.1433. The plot of Cattell's scree is presented on Figure 2.

The Cattell's scree plot suggests to take factors Z_1, Z_2 and Z_3 into consideration. The same factors explain sufficient percentage of changeability of all observed variables. Group of factors (Z_1, Z_2, Z_3) explains 93.25% of changeability of combustion heat, 92.41% of ash contents, 94.32% of sulfur contents, 90.33% of volatile parts contents and 97.99% of mois-

Tab. 10. Ash contents by separation in accordance to particle density – coal, type 31

Tab. 10. Zawartość popiołu według rozdziału na frakcje gęstościowe – węgiel, typ 31

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
1.250	2.810	0.000	5.940	1.260	1.600	1.550	1.840
1.350	5.940	2.810	17.320	9.000	8.910	9.070	8.980
1.450	17.320	5.940	26.090	16.760	16.270	16.650	16.230
1.550	26.090	17.320	32.490	24.500	24.270	24.290	24.230
1.650	32.490	26.090	38.350	32.250	37.400	31.940	32.100
1.750	38.350	32.490	48.380	39.990	40.090	39.490	39.600
1.850	48.380	38.350	0.000	47.740	47.740	48.470	48.440

Tab. 11. Ash contents by separation in accordance to particle size – coal, type 34.2

Tab. 11. Zawartość popiołu według rozdziału na klasy ziarnowe – węgiel, typ 34.2

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
0.750	7.110	0.000	8.670	7.600	6.970	7.480	6.980
2.075	8.670	7.110	10.280	8.620	8.940	8.550	8.900
4.725	10.280	8.670	12.950	10.660	10.840	10.660	10.820
7.150	12.950	10.280	14.940	12.530	12.590	12.580	12.600
9.000	14.940	12.950	15.830	13.960	14.170	14.030	14.170
11.250	15.830	14.940	16.970	15.690	15.720	15.790	15.750
13.250	16.970	15.830	18.480	17.230	17.230	17.370	17.270
15.000	18.480	16.970	20.550	18.580	18.530	18.750	18.580
18.000	20.550	18.480	0.000	20.890	20.620	20.530	20.550

Tab. 12. Ash contents by separation in accordance to particle density – coal, type 34.2

Tab. 12. Zawartość popiołu według rozdziału na frakcje gęstościowe – węgiel, typ 34.2

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
1.250	1.270	0.000	4.280	0.310	0.210	0.750	0.640
1.350	4.280	1.270	16.380	7.310	7.360	7.500	7.540
1.450	16.380	4.280	24.520	15.090	15.250	14.660	14.820
1.550	24.520	16.380	29.800	22.870	22.860	22.130	22.110
1.650	29.800	24.520	34.790	30.650	30.580	29.630	29.560
1.750	34.790	29.800	49.090	38.440	38.400	36.140	35.860
1.850	49.090	34.790	0.000	46.220	46.240	49.300	49.310

Tab. 13. Ash contents by separation in accordance to particle size – coal, type 35

Tab. 13. Zawartość popiołu według rozdziału na klasy ziarnowe – węgiel, typ 35

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
0.750	16.250	0.000	22.340	20.580	16.380	19.240	16.520
2.075	22.340	16.250	32.180	23.170	24.320	22.850	23.910
4.725	32.180	22.340	35.480	28.340	29.240	28.370	29.070
7.150	35.480	32.180	37.080	33.060	35.500	33.280	35.110
9.000	37.080	35.480	41.310	36.670	38.530	37.050	38.500
11.250	41.310	37.080	45.300	41.060	41.220	42.140	41.770
13.250	45.300	41.310	49.620	44.950	44.760	46.740	45.650
15.000	49.620	45.300	50.840	48.360	47.990	50.050	48.990
18.000	50.840	49.620	0.000	54.210	52.470	50.880	50.870

ture, while factor Z_1 is related to variables X_1, X_2, X_3, X_4; factor Z_2 to variables X_2, X_3, X_4 and factor Z_3 to variable X_5.

Another criterion of limiting number of factors is determination of amount of percent of total variance explained by chosen factors (most often it is required to not be lower than 85%). In this case, for coal type 31, factors Z_1 and Z_2 explain 93.14% of variation of variable X_1 (combustion heat), 96.65% of variation of variable X_2 (ash contents), 99.00% of variation of variable X_3 (sulfur contents), 91.14% of variation of variable X_4 (volatile parts contents) and 89.14% of variation of variable X_5 (analytic moisture). For coal type 34.2, factors Z_1, Z_2 and Z_3 explain 95.21% of variation of variable X_1, 97.48% of variation of variable X_2, 99.95% of variation of variable X_3, 86.72% of variation of variable X_4 and 99.68% of variation of variable X_5. Finally, for coal type 35, these factors explain 98.21% of variation of variable X_1, 98.39% of variation of variable X_2, 99.87% of variation of variable X_3, 95.57% of variation of variable X_4 and 99.00% of variation of variable X_5.

The influences of individual factors on considered variables in all fractions of individual types of coal are presented in Tables 3-8. It was assumed that changeability of each feature should be explained by factors in at least 85%.

3.2. Mathematical modeling

On the basis of one- and multidimensional regressive analysis four models presenting relations between ash contents in certain particle size fraction (or density fraction), particle density (or particle size) and ash contents in neighboring size or density fractions.

The general form of proposed models are:

Tab. 14. Ash contents by separation in accordance to particle density – coal, type 35
Tab. 14. Zawartość popiołu według rozdziału na frakcje gęstościowe – węgiel, typ 35

x_1 [mm]	y [%]	x_2 [%]	x_3 [%]	y_1 [%]	y_2 [%]	y_3 [%]	y_4 [%]
1.250	2.370	0.000	8.290	0.810	1.770	0.710	1.710
1.350	8.290	2.370	15.160	9.290	9.030	9.230	8.850
1.450	15.160	8.290	27.700	17.770	17.100	17.810	17.070
1.550	27.700	15.160	35.540	26.250	25.400	26.350	25.450
1.650	35.540	27.700	43.360	34.730	34.980	34.880	35.550
1.750	43.360	35.540	51.340	43.210	43.480	43.410	43.980
1.850	51.340	43.360	0.000	51.690	51.990	51.370	51.340

Tab. 15. Values of errors by particle size as partition feature
Tab. 15. Wartości błędów według wielkości ziarna jako cechy rozdziału

	Coal, type 31					Coal, type 34.2				Coal, type 35			
	y_1	y_2	y_3	y_4		y_1	y_2	y_3	y_4	y_1	y_2	y_3	y_4
R^2	0.902	0.919	0.938	0.941		0.990	0.993	0.991	0.993	0.952	0.982	0.972	0.987
MSE	5.190	5.100	4.460	4.790		0.498	0.200	0.490	0.480	2.760	1.840	2.260	1.720
Cp	4.220	3.810	2.210	4.000		2.540	2.280	3.490	4.000	12.290	3.910	4.480	4.000

Tab. 16. Values of errors by particle density as partition feature
Tab. 16. Wartości błędów według gęstości ziarna jako cechy rozdziału

	Coal, type 31				Coal, type 34.2				Coal, type 35			
	y_1	y_2	y_3	y_4	y_1	y_2	y_3	y_4	y_1	y_2	y_3	y_4
R^2	0.990	0.990	0.990	0.991	0.979	0.979	0.987	0.988	0.994	0.996	0.994	0.995
MSE	1.870	2.060	2.030	2.310	2.710	3.020	2.300	2.610	1.620	1.620	1.790	1.770
Cp	0.270	2.200	2.080	4.000	2.400	4.340	2.110	4.000	1.175	2.343	3.100	4.000

- One-dimensional model

$y = ax_1 + b$ (4)

- Two-dimensional models

$y = a_1x_1 + a_2x_2 + b$ (5)

and

$y=a_1x_1 + a_2x_3+ b$ (6)

- Three-dimensional model

$y=a_1x_1 + a_2x_2 + a_3x_3 + b$ (7)

where:
y – ash contents in certain particle size (or particle density) fraction;
x_1 – particle size or particle density;
x_2 – ash contents in previous particle size (or density) fraction;
x_3 – ash contents in following particle size (or density) fraction.

Because of the fact that during material separation process particles from other fractions transfer to the certain considered fraction in two- and three-dimensional models ash contents in neighboring fractions were taken into account and their influence was evaluated.

The analysis was conducted for all three types of coal. The results of analyzes were presented in Tables 9–14.

On the basis of the results the regressive analysis was conducted and the following models were obtained in accordance to the equations (4)–(7):

$y_1 = 2.505x_1 + 10.910$
$y_2 = 1.397x_1 + 0.385x_2 + 10.307$
$y_3 = 2.407x_1 + 0.166x_2 + 6.480$
$y_4 = 1.979x_1 + 0.153x_2 + 0.145x_3 + 6.720$

Individual models describing formulas presented in equations (4)–(7) are in this case as following:

$y_1 = 77..470x_1 - 95.575$
$y_2 = 70.270x_1 + 0.103x_2 - 86.240$
$y_3 = 77.960x_1 - 0.024x_2 - 95.750$
$y_4 = 71.590x_1 + 0.091x_2 - 0.023x_3 - 87.509$

In this case, functions describing relations presented in equations (4)–(7) are in form:

$y_1 = 0.771x_1 + 7.022$
$y_2 = 0.618x_1 + 0.162x_2 + 6.509$
$y_3 = 0.770x_1 - 0.027x_2 + 6.682$
$y_4 = 0.627x_1 + 0.152x_2 - 0.007x_3 + 6.453$

The individual functions describing relations given by equations (4)–(7) are in this case as following:

$y_1 = 77.822x_1 - 97.748$
$y_2 = 79.838x_1 - 0.031x_2 - 100.381$
$y_3 = 80.180x_1 - 0.105x_2 - 99.928$
$y_4 = 82.123x_1 - 0.030x_2 - 0.105x_3 - 101.565$

This time the form of the equations (4)–(7) is as following:

$y_1 = 1.949x_1 + 19.127$
$y_2 = 0.895x_1 + 0.416x_2 + 15.706$
$y_3 = 1.955x_1 + 0.104x_2 + 15.454$
$y_4 = 1.118x_1 + 0.329x_2 + 0.057x_3 + 14.409$

In this case the models created in accordance to equations (4)–(7) are in form:

$y_1 = 84.798x_1 - 105.185$
$y_2 = 67.294x_1 + 0.227x_2 - 82.350$
$y_3 = 84.577x_1 + 0.010x_2 - 105.096$
$y_4 = 63.617x_1 + 0.268x_2 + 0.022x_3 - 77.993$

3.3. Investigation of models quality

To evaluate the quality of models obtained by means of general formulas presented in equations (4)–(7) such factors as R^2 coefficient, mean squared error *MSE* and Mallow's statistics C_p were calculated which are given by the following formulas (Stanisz, 2007; Tumidajski and Saramak, 2009), presented in equations (8)–(10):

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_{ki} - y_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}, k = 1,2,3,4 \quad (8)$$

$$MSE = \sqrt{\frac{\sum_{i=1}^{n}(y_{ki} - y_i)^2}{n - q - 1}} \quad (9)$$

where q is an amount of independent variables occurring in considered function

$$C_p = \frac{\sum_{i=1}^{n}\left(y_{ki} - y_i\right)^2}{MSE_4^2} \quad (10)$$

where MSE4 is mean squared error calculated for y_4.

The obtained results of calculated errors are presented in Tables 15 and 16.

4. Conclusions

Because of the fact that the most often three factors occur in individual fractions and considering power of relations between individual properties the investigated variables can be divided into three subsets. First one contains combustion heat, ash contents and volatile parts contents, second one contains sulfur contents and the third one contains moisture. In scientific works [3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24], through application of various visualization methods it was claimed that features being sufficient to identify coal type are sulfur contents, moisture and volatile parts contents. The conducted analysis confirms these results. The selection of variable X_4 (volatile parts contents) occurs from the fact that this variable is explained by other factor than mutual factor with variables moisture and combustion heat.

Considering the mathematical models it must be said that during grained material separation (in this case – coal) into particle size or density fractions some of the particles from neighboring fractions (j-1 or j+1) occur in jth fraction it seems to be justified to consider this fact during construction of mathematical model describing ash contents by means of particle size or density.

In the paper four models are proposed:

- One-dimensional, which does not consider influences of neighboring fractions;
- Two-dimensional, which takes the influence of one of neighboring fractions into consideration – two models of such type;
- Three-dimensional, which takes the influence of both neighboring fractions.

The verification of these models was conducted on the basis of three factors: R^2 coefficient, mean squared error *MSE*- and Mallow's statistics C_p.

Taking into consideration the R^2 coefficient it is visible that for all considered models the value of this factor is relatively high (above 0.9). It can be noticed that the R^2 achieves higher values when the separation is done in accordance to particle density than in case of particle size (apart from coal, type 34.2).

Furthermore, the value of mean squared error indicates that the models are well fitted, but (apart from coal, type 34.2) significantly better fitting to empirical results is achieved in case of separation done in accordance to particle density. To compare the models for various dimensions the Mallow's statistics C_p was used, which suggests that the best model is the one which values of C_p is close to the value q+1, where q is a number of independent variables occurring in the model. Analyzing Tables 8 and 9 it can be stated that the best model is a three-dimensional one, but in some cases, as for coal, type 35 by separation done in accordance to particle size, the two-dimensional models have the value of C_p around q+1=3.

The analyzed cases indicate that despite satisfying results of one-dimensional approximation to obtain better models is worthy to consider also influences of the researched feature in neighboring fractions.

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Analiza czynnikowa i modelowanie matematyczne w określeniu jakości węgla

Wykonano rozdział trzech typów węgla o różnych charakterystykach, pochodzących z trzech różnych kopalni węgla kamiennego w Polsce (typy 31, 34.2 oraz 35, według Polskich norm, którymi były węgiel energetycznym, pół-koksujący oraz koksujący) na klasy ziarnowe a następnie na frakcje gęstościowe. Dla każdej otrzymanej w ten sposób frakcji wielkościowo-gęstościowej zmierzono następujące parametry: ciepło spalania, zawartość popiołu, zawartość siarki, zawartość części lotnych, wilgotność analityczna. W ten sposób otrzymano siedmiowymiarowy wektor danych. Za pomocą analizy czynnikowej wybrano istotne cechy węgla, które decydują o jego przynależności do określonego typu węgla. Aby ocenić prawidłowość zastosowanej metody wykorzystano test sferyczności Bartletta oraz współczynnik Kaisera-Mayera-Olkina (KMO). Otrzymane wyniki porównano z wynikami otrzymanymi w poprzednich pracach, które uzyskano metodą tuneli obserwacyjnych. Wyniki pokazały, które cechy węgla są niezbędne do określenia typu węgla, co wpływa na dobór odpowiedniej metody jego wzbogacania. Ponadto, zaprezentowano model prezentujący relacje pomiędzy tymi cechami a wielkością i gęstością ziaren. Ponieważ ziarna określonej wielkości lub gęstości mogą występować w sąsiednich klasach lub frakcjach, wykonano trzy typy modeli, bazując na analizie regresji. Analiza została wykonana dla trzech typów węgli. Ponieważ modele zawierają różne ilości zmiennych niezależnych do oceny i porównania otrzymanych wyników zastosowano współczynnik determinacji R^2, błąd średniokwadratowy (MSE) oraz statystykę Mallowa C_p.

Słowa klucze: *węgiel, wielowymiarowa analiza statystyczna, analiza czynnikowa, jakość węgla, wielkość ziarna, gęstość ziarna*