The Method for Assessing the Impact of Variable Coal Demand on the Efficiency of Mine Operations

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Abstract

The method presented in the article is based on Monte Carlo simulation and involves studying the impact of random demand fluctuations on the efficiency of mines and mine groups (companies). For random demand fluctuations, a normal distribution is assumed, and the analysis variants present-ed include:

- *Adopting the mean and variance values based on retrospective data,*
- *Considering the most probable forecast error resulting from predictive formulas,*
- *Taking into account correlated changes in demand.*

The results obtained are presented in the form of histograms of the degree of operational lever-age. These histograms allow for predicting how the degree of operational leverage of mines will de-velop, as well as estimating the direction and probability of these changes. The developed and veri-fied sensitivity analysis using real examples constitutes a useful element in rationalizing decision-making processes.

Keywords: *sensitivity analysis, degree of operating leverage, the SIMPLEX algorithm, the Monte Carlo method*

1. INTRODUCTION

Changes in the level of coal requirement have a significant effect on the financial situations of both individual mines and groups. Therefore, in market conditions it is essential to carry out multivar-iant analyses to assess the sensitivity of coal production and sales plans, as well as other economic and technical quantities on changes in requirements.

The present author's research [5, 6, 7] on the options for sensitivity analysis of coal production and sales plans, the product structure, reserves, capacity and so on to changes in demand have re-vealed the significant suitability of the Monte Carlo method in the unbalanced Polish coal market [3, 4, 12, 14].

The sensitivity analysis using the Monte Carlo method presented in this paper covers research on the effect of random fluctuations in demand on the profit and the degree of operating leverage on the basis of a real coal mines.

2. THE ESSENCE OF OPERATING LEVERAGE

Leverage (in terms of finance) is used when changing the values of certain economic quantities causes more than proportionate change in other economic quantities.

Any increase (decrease) in gross receipts from sales will bring the company a more than pro-portional increase (decrease) in gross profit on sales (percentage-wise) - assuming constancy in other factors which affect its level. This is called operating leverage. In order to determine what change in profit will be accompanied by a specific gain in sales, the degree of operating leverage (DOL) is cal-culated [13]:

$$DOL = \frac{\%\Delta EBIT}{\%\Delta S} \tag{1}$$

or

$$DOL = \frac{S_o - Kz_o}{EBIT_o} \tag{2}$$

where:
%ΔEBIT – Percentage increase in profit before interest and taxes,
%ΔS – Percentage growth in net sales,
S_o – The value of net sales as of the base, (PLN),
Kz_o – The level of variable costs as of the base, (PLN),
$EBIT_o$ – The level of profit before interest and taxes as of the base, (PLN).

The operating leverage mechanism is a useful tool in the ongoing management of a company. With it, the rate of change in profit can be determined, for example: with an increase (decrease) sales, for example, of 10%, the profit made by the company will increase (decrease) by 10% × DOL. The degree of operating leverage (DOL) will depend on the profitability of sales and the cost structure taking into account their variability. Its size varies depending on the level of sales, which is the basis for the calculations. Hence, the operating leverage is used, inter alia, for predicting a company's fu-ture economic performance [13].

3. CHARACTERISTICS OF THE PROPOSED METHOD

The basis for an analysis of profit sensitivity and the degree of operating leverage to changes in demand is the set of optimal solutions for optimising production and coal sales for the mining com-pany. The Monte Carlo method is used for this. The set of optimal solutions for optimisation is creat--ed by repeated calculation of an optimal programme for the production and sale of coal with a given, random, demand scenario. The optimal solution, however, is obtained using the SIMPLEX algorithm.

The analysis is conducted for a set of 1 000 random demand sets. The demand vector drawn is the subvector of the right sides of the 4 optimisation model equation [7]:

Tab. 1. Technical and economic coefficients for mines „B" and „D". Source: Own preparation
Tab. 1. Wskaźniki techniczno-ekonomiczne kopalń „B" i „D". Źródło: przygotowanie własne

Specification	Unit	Mine „B"	Mine „D"
Average Extraction	ton/day	11,800	12,000
Max. Extraction	netto ton	2,700,000	4,600,000
Unit cost	PLN/t	159.02	115.87
Fixed cost	%	86.96	86.98

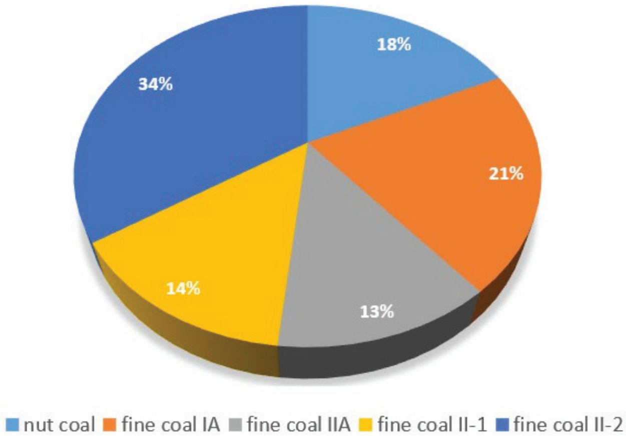

Fig. 1. Assortment structure of production for mine „B". Source: Own preparation
Rys. 1. Struktura asortymentowa produkcji kopalni „B"

Objective function:

$$\sum_{j=1}^{p}\sum_{i=1}^{r_j}\sum_{k=1}^{m_{ij}}(c_{ijk}-kz_{ijk})\cdot x_{ijk}-\sum_{j=1}^{p}Ks_j \rightarrow \max \qquad (3)$$

Sales restrictions:

$$\sum_{j=1}^{p}\sum_{i=1}^{rj}\sum_{k=1}^{mij}x_{ijk}\le Z_k \qquad \text{for all k} \qquad (4)$$

where:
x_{ijk} – net amount of extracted coal of *ij* type accepted by consumers in group *k*, (netto tone),
c_{ijk} – price of *ij* type of coal,
kz_{ijk} – variable cost for mine *j*,
Ks_j – fixed cost for mine *j*,
Z_k – consumer demand for group *k*,
i – index of coal type, *i* =1, 2, 3, ...r_j,
j – index of mine, *j* =1,2,3,... *p*,
k_n – index of consumer groups, *k* = 1,2,3,..., m_{ij}, where m_{ij} marks numerousness miscellany k_n for coal of *ij* type.

The remaining restrictions in the model relate to the structure of production and the capacity of individual mines [7]. The reality of the solutions obtained is assured by allowing the possibility of storing coal.

Each of the randomly selected demand elements is a random variable with normal distribution. The projected volume of demand is adopted as the expected value (nominal), and the most likely forecast error resulting from the predictive formulae is adopted as the dispersion (standard devia-tion).

As a result of the test, new optimal solutions are obtained when considering different demands (new values for the objective function). This makes it possible to present the results of the analysis in the form of a histogram of profit and other economic and technical quantities for both the entire com-pany and individual mines, as well as determining the likelihood of obtaining the assumed level for the analyzed quantities for the company and the mines comprising it.

4. EXAMPLE OF CALCULATIONS AND EVALUATION OF RESULTS

Calculations are performed for a selected mine „D", which is part of a coal company compris-ing seven mines with different production characteristics. In Table 1, the production capacity of the analyzed „B" and „D" mines is shown, together with technical and economic indicators, and their the product structure is illustrated in Figures 1 and 2.

In the first stage of the analysis, the magnitude of demand for each group of consumers was randomly generated according to a normal distribution with the expected value (nominal) equal to the planned demand (sales). For retrospective data, the best model for each group of consumers was fit-ted using regression analysis, and based on this, the forecasted demand for the planned forecast year was determined. The adopted dispersion value (σ_r) represents the standard error of estimation of the regression function, which is a measure indicating the average deviations of actual values of the de-pendent variable (coal demand of consumers) from the theoretical values of this variable calculated from the regression function. It is one of the parameters of the random component distribution that allows us to infer the goodness of fit of the model to the available empirical data. It is calculated using the formula [1, 2, 8, 9, 10, 11]:

$$\sigma_r^2 = \frac{\sum_{n=1}^{N}\left(y_n - y_{\text{mod}}\right)^2}{N-K}, \qquad (5)$$

Table 2 compares the annual nominal values and dispersions in demand for individual groups of customers. Table 3, meanwhile, presents the optimal production and sales plan for „B" and „D" mines.

For various coal products, the following sales prices were chosen:

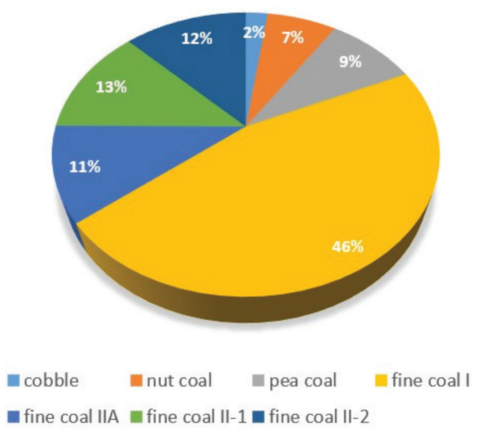

Fig. 2. Assortment structure of production for mine „D". Source: Own preparation
Rys. 2. Struktura asortymentowa produkcji kopalni „D"

Tab. 2. Nominal value and dispersion σ_{yprog} for every group of consumers]. Source: Own preparation
Tab. 2. Zestawienie wartości nominalnej zapotrzebowania oraz dyspersji σ_r dla poszczególnych grup odbiorców. Źródło: przygotowanie własne

Name of consumer group	Nominal prognosis values (t)	Dispersion σ_r (t)
Export 8	300,857	95,728
Export 9	419,447	133,461
Indv. consumers 1	336,060	13,035
Indv. consumers 3	5,475,600	212,387
Indv. consumers 4	1,391,200	53,962
Dust kettles	2,385,300	92,521
Grates 2	265,940	10,315
Grates 3	1,095,000	42,472
Grates 4	567,619	22,017
Chamber grates 1	425,765	16,514

- cobble – 450 PLN/t,
- nut coal – 381 PLN/t,
- pea coal – 365 PLN/t,
- fine coal I – 324 PLN/t,
- fine coal IA – 321 PLN/t,
- fine coal IIA – 310 PLN/t,
- fine coal II – 300 PLN/t.

The variable unit cost was estimated at 40 PLN/t.

The use of the Monte Carlo method for sensitivity analysis of coal production and sales plans to changes in demand involves multiple iterations to determine the optimal production and sales pro-gram for mines under assumed random demand scenarios.The selected number of 1 000 draws ena--bles an adequate set of production tasks and corresponding financial results for the various mines to be obtained, as well as the calculation of the degree of operating leverage.

The obtained results are presented in Table 6 and illustrated in Figures 3-10. The vertical black line represents the value of the operational leverage resulting from the optimal plan for each mine.

In the second stage, an analysis of the effects of random changes in demand was conducted based on the most probable forecast error. To estimate the forecast error of demand for each consum-er, the following formula was used [1, 2, 8, 9, 10, 11]:

$$\sigma_{yprog} = \sqrt{\sigma_r^2 + \sigma_{\hat{y}}^2} \qquad (6)$$

where:
σ^2_r – variation in the remainder factor, defining according to the following formula:

$$\sigma_r^2 = \frac{\sum_{n=1}^{N} \left(y_n - y_{\mathrm{mod}}\right)^2}{N-K}, \qquad (7)$$

where:
y_n – actual value of endogenous factor,
y_{mod} – model-based value of endogenous factor,
N – number of observations,
K – number of estimating parameters for model structure.
$\sigma^2_{\hat{y}}$ – Estimation of variance in prognosis model:

$$\sigma_{\hat{y}}^2 = \left[1 \quad x_{N+2}\right] \cdot \left[X^T \cdot X\right]^{-1} \cdot \left[1 \quad x_{N+2}\right]^T \cdot \sigma_r^2, \qquad (8)$$

where:
x_{N+2} – time, during which prognosis is prepared.
also

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ 1 & x_N \end{bmatrix}, \qquad X^T X = \begin{bmatrix} N & \sum_{n=1}^{N} x_n \\ \sum_{n=1}^{N} x_n & \sum_{n=1}^{N} x_n^2 \end{bmatrix} \qquad (9)$$

The nominal values and dispersions of consumer demand used in this stage of the analysis are presented in Table 4. The obtained results are shown in Table 6 and illustrated in Figures 3-10.

In the third stage, the impact of correlated fluctuations in demand on the operational leverage of individual mines was examined. To do this, the nominal value of the demand forecast for each consumer was decreased (P1) by the value of the model error in one case and increased (P2) by the value of the model error in another case (Formula 6). The data is

Tab. 3. The optimal plans of production and sales for "B" and „D" mines. Source: Own preparation

Tab. 3. Optymalny plan produkcji i sprzedaży węgla dla kopalń „B" i „D". Źródło: przygotowanie własne

Mine "B"		
Offer: 2,700,000 t Sold: 1,776,600 t	Gross profits: 62,938,398 PLN Mine reserves: 0 t	
Consumers	**Coal assortment**	**Quantity (t)**
Indv. consumers 4	nut coal	328,143
Export 8	nut coal	14,857
Indv. Consumers 1	nut coal	143,000
Indv. consumers 3	fine coal IA	237,000
Dust kettles	fine coal IA	330,000
Dust kettles	fine coal II	380,700
Export 9	fine coal II	342,900
Coal dump	fine coal II	923,400
Mine "D"		
Offer: 4,600,000 t Sold: 4,022,220 t	Gross profits: 371,140,074 PLN Mine reserves: 0 t	
Consumers	**Coal assortment**	**Quantity (t)**
Indv. consumers 4	cobble	101,200
Indv. consumers 4	nut coal	322,000
Indv. consumers 4	pea coal	414,000
Indv. consumers 3	fine coal I	2,120,600
Grates 4	fine coal II	567,620
Indv. consumers 3	fine coal IIA	496,800
Coal dump	fine coal II	7,380
Coal dump	fine coal II	570,400

Tab. 4. Nominal value and dispersion σ_{yprog} for every group of consumers [t]. Source: Own preparation

Tab. 4. Zestawienie wartości nominalnej zapotrzebowania oraz dyspersji σ_{yprog} dla poszczególnych grup odbiorców. Źródło: przygotowanie własne

Name of consumer group	Nominal prognosis values (t)	Dispersion σ_{yprog} (t)
Export 8	300,857	70,205.89
Export 9	419,447	116,913.60
Indv. consumers 1	336,060	10,565.02
Indv. consumers 3	5,475,600	140,552.80
Indv. consumers 4	1,391,200	35,710,60
Dust kettles	2,385,300	61,228.09
Grates 2	265,940	6,826.40
Grates 3	1,095,000	28,107.22
Grates 4	567,619	14,570.20
Chamber grates 1	425,765	10,928.80

Tab. 5. The compilation of the nominal forecast values P1 and P2, as well as the dispersions σ_{yprog} for each con-sumer group. Source: Own preparation

Tab. 5. Zestawienie wartości nominalnej prognoz P1 i P2 oraz dyspersji σ_{yprog} dla poszczególnych grup odbiorców. Źródło: przygotowanie własne

Name of consumer group	Nominal prognosis values P_1 (t)	Nominal prognosis values P_2 (t)	Dispersion σ_{yprog} (t)
Export 8	230,651.11	371,062.89	70,205.89
Export 9	302,533.40	536,360.60	116,913.60
Indv. consumers 1	325,494.98	346,625.02	10,565.02
Indv. consumers 3	5,335,046.20	5,616,151.80	140,552.80
Indv. consumers 4	1,355,489.40	1,426,910.60	35,710.60
Dust kettles	2,324,071.91	2,446,528.10	61,228.09
Grates 2	259,113.60	272,766.40	6,826.40
Grates 3	1,066,892.78	1,123,107.20	28,107.22
Grates 4	553,048.80	582,189.20	14,570.20
Chamber grates 1	414,836.20	436,693.80	10,928.80

presented in Table 5, and the ob-tained results are shown in Table 6 and illustrated in Figures 3-10

Analyzing the presented histograms (Figures 3-10) of the achieved operational leverage degree for mines "B" and "D," their distortion can be observed. This distortion results from the mismatch between the production structure of the analyzed mines and the demand from consumers, as con-firmed in previous studies [1].

The nominal value of the operational leverage degree for mine "B" is 6.52 (represented by the black vertical line). This means that a decrease in sales, for example, by 10%, will result in a profit decrease of 65.2%, and vice versa. The probability of maintaining such a leverage degree is 0.80 and 0.90, respectively, for σ_r and σ_{yprog} (Figures 3 and 4). With a decrease in demand to the P_1 level, the probability of achieving the nominal leverage value will be approximately 50% (Figure 5), while in the case of an increase in consumer demand to the P_2 level, it will be only 13% (Figure 6). In each analyzed case, there is a higher probability of a favorable situation for the mine, namely a decrease in the degree to the mean value. For σ_r, the DOL will be 6.28 (89% chance), for σ_{yprog}, the DOL will be 6.41 (92% chance), and for P_1 and P_2, respectively, the DOL will be 6.19 (70% chance) and 6.29 (90% chance). Other values of the leverage degree may occur, but with a very low probability ranging from 0.001 to 0.129 (Table 6).

For mine "D," the probability of achieving the nominal level of the leverage degree (represent-ed by the black vertical line) is almost equal to 1 in each analyzed case, corresponding to a level of 2.16 (Figures 7-10). There are only 3 chances out of 1000 for the value 2.27 to occur (for σ_r and P_1) (Figure 2, Table 4).

5. CONCLUSION

The magnitude of the demand from potential and existing customers has a decisive impact on the mine's production volume, and thus the effectiveness of the company. The presented method of analysis provides a useful tool in assisting decision making, particularly in the area of reasonable volumes for production and sale of coal and also when conducting specific strategies regarding the mine's (company's) continued operation. It also provides the basis for action to

Tab. 6. The compilation of nominal, minimum, maximum, and mean values of the predicted operational lever-age, as well as the probability of achieving these values for the nominal values P1 and P2, and dispersions σ_r and σ_{yprog}. Source: Own preparation

Tab. 6. Zestawienie wartości nominalnej, minimalnej, maksymalnej i średniej przewidywanego stopnia dźwigni operacyjnej oraz prawdopodobieństwa jego osiągnięcia dla wartości nominalnych P1 i P2 oraz dyspersji σ_r, σ_{yprog}. Źródło: przygotowanie własne

	Degree of operating leverage (-)				**Likelihood of attaining (-)**			
	Nominal value	Minimum value	Maximum value	Average value	Minimum value	Maximum value	Nominal value	Average value
Mine „B”								
σ_r	6.52	2.19	7.53	6.28	0.001	0.008	0.805	0.886
σ_{yprog}	6.52	3.73	6.53	6.41	0.003	0.003	0.902	0.917
P_1	6.52	3.58	6.91	6.19	0.003	0.054	0.523	0.696
P_2	6.52	2.80	6.52	6.29	0.003	0.129	0.129	0.900
Mine „D”								
σ_r	2.164	2.16	2.26	2.169	0.115	0.003	0.885	0.454
σ_{yprog}	2.164	2.16	2.18	2.166	0.003	0.003	0.586	0.437
P_1	2.164	2.16	2.27	2.17	0.006	0.003	0.892	0.514
P_2	2.164	2.16	2.18	2.162	0.446	0.003	0.156	0.994

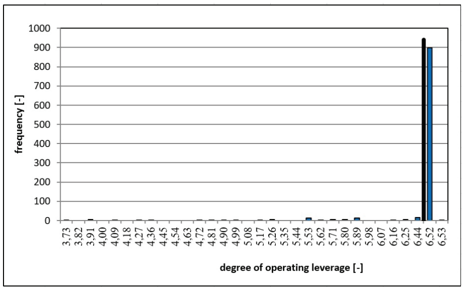

Fig. 3. Histogram showing frequencies of achieving given degree of operating leverage for mine „B” with dispersion σr . Source: Own preparation

Rys. 3. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „B” przy dyspersji σr. Źródło: przygotowanie własne

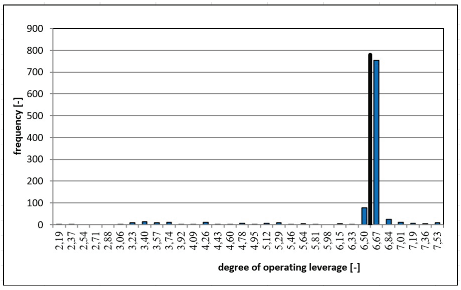

Fig. 4. Histogram showing frequencies of achieving given degree of operating leverage for mine „B" with dispersion σ_{yprog}. Source: Own preparation

Rys. 4. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „B” przy dyspersji σ_{yprog}. Źródło: przygotowanie własne

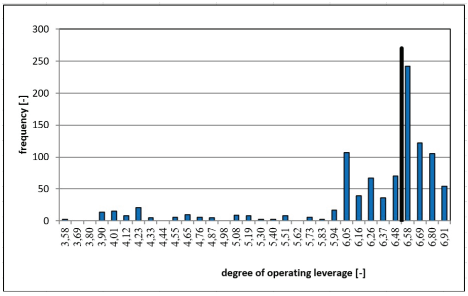

Fig. 5. Histogram of the frequency of obtained operational leverage degrees for mine “B" with the nominal value P1 and dispersion σ_{yprog}

Rys. 5. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „B” przy wartości nominalnej P1 i dyspersji σ_{yprog}. Źródło: przygotowanie własne

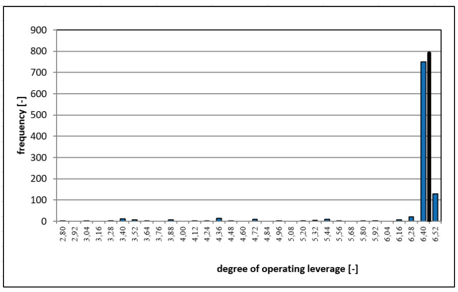

Fig. 6. Histogram of the frequency of obtained operational leverage degrees for mine "B" with the nominal value P_2 and dispersion σ_{yprog}

Rys. 6. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „B” przy wartości nominalnej P_2 i dyspersji σ_{yprog}

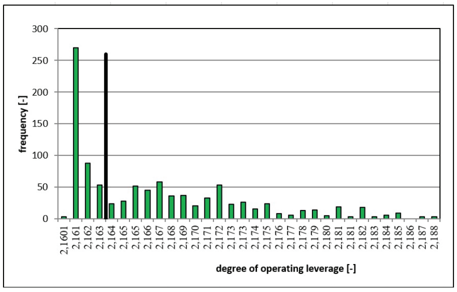

Fig. 7. Histogram showing frequencies of achieving given degree of operating leverage for mine „D" with dispersion σ_r. Source: Own preparation
Rys. 7. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „D" przy dyspersji σ_r. Źródło: przygotowanie własne

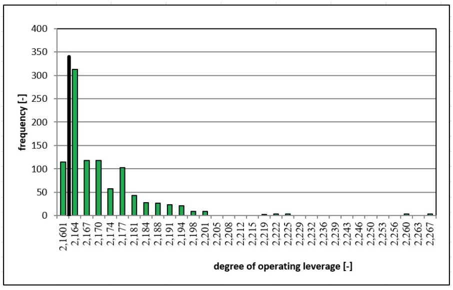

Fig. 8. Histogram showing frequencies of achieving given degree of operating leverage for mine „D" with dispersion σ_{yprog}. Source: Own preparation
Rys. 8. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „D" przy dyspersji σ_{yprog}. Źródło: przygotowanie własne

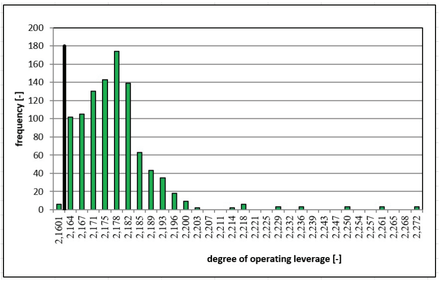

Fig. 9. Histogram of the frequency of obtained operational leverage degrees for mine "D" with the nominal value P1 and dispersion σ_{yprog}
Rys. 9. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „D" przy wartości nominalnej P1 i dyspersji σ_{yprog}

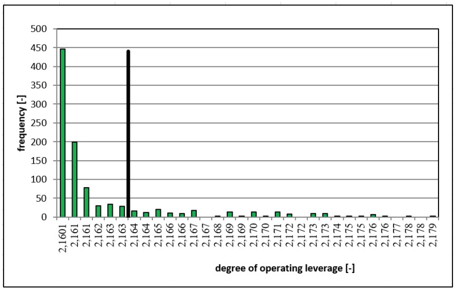

Fig. 10. Histogram of the frequency of obtained operational leverage degrees for mine "B" with the nominal value P2 and dispersion σ_{yprog}
Rys. 10. Histogram częstości uzyskiwanych stopni dźwigni operacyjnej dla kopalni „D" przy wartości nominalnej P2 i dyspersji σ_{yprog}

adapt the production structure, both in terms of quantity and quality, to customer requirements.

This method of analysis is likely to reflect real situations that might occur. The results obtained for the multi-variant change in demand enable a direct indication of what values the volumes ana-lysed can achieve (for example: profit or the degree of operating leverage), and with what probability.

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Metoda oceny wpływu zmiennego zapotrzebowania odbiorców węgla na efektywność funkcjonowania kopalń

Zaprezentowana w artykule metoda oparta jest na symulacji Monte Carlo i obejmuje badanie wpływu wahań losowych zapotrzebowania na efektywność kopalń oraz ich grup (spółek). Dla losowych wahań zapotrzebowania przyjęto rozkład normalny, a przedstawione warianty analizy uwzględniają:

- *przyjęcie wartości oczekiwanej i dyspersji według danych retrospektywnych;*
- *przyjęcie najbardziej prawdopodobnego błędu prognozy wynikającego z formuł predykcyjnych;*
- *uwzględnienie skorelowanych zmian zapotrzebowania.*

Uzyskane wyniki przedstawiono w postaci histogramów stopnia dźwigni operacyjnej. Pozwalają one przewidywać, jak będzie kształtował się stopień dźwigni operacyjnej kopalń, jak również umożliwia oszacować, w którym kierunku zmiany te będą postępować i z jakim prawdopodobieństwem. Opracowana i zweryfikowana na realnych przykładach analiza wrażliwości stanowi przydatny element racjonalizacji procesów decyzyjnych.

Słowa kluczowe: *analiza wrażliwości, dźwignia operacyjna, algorytm SIMPLEX, metoda Monte Carlo*